Measurement of the properties of materials

useful in the design

of mechanical resonators.

Modal Mechanics

1. Density, ρ = *mass/unit volume*

2. Young's modulus, E = *stress/unit strain*

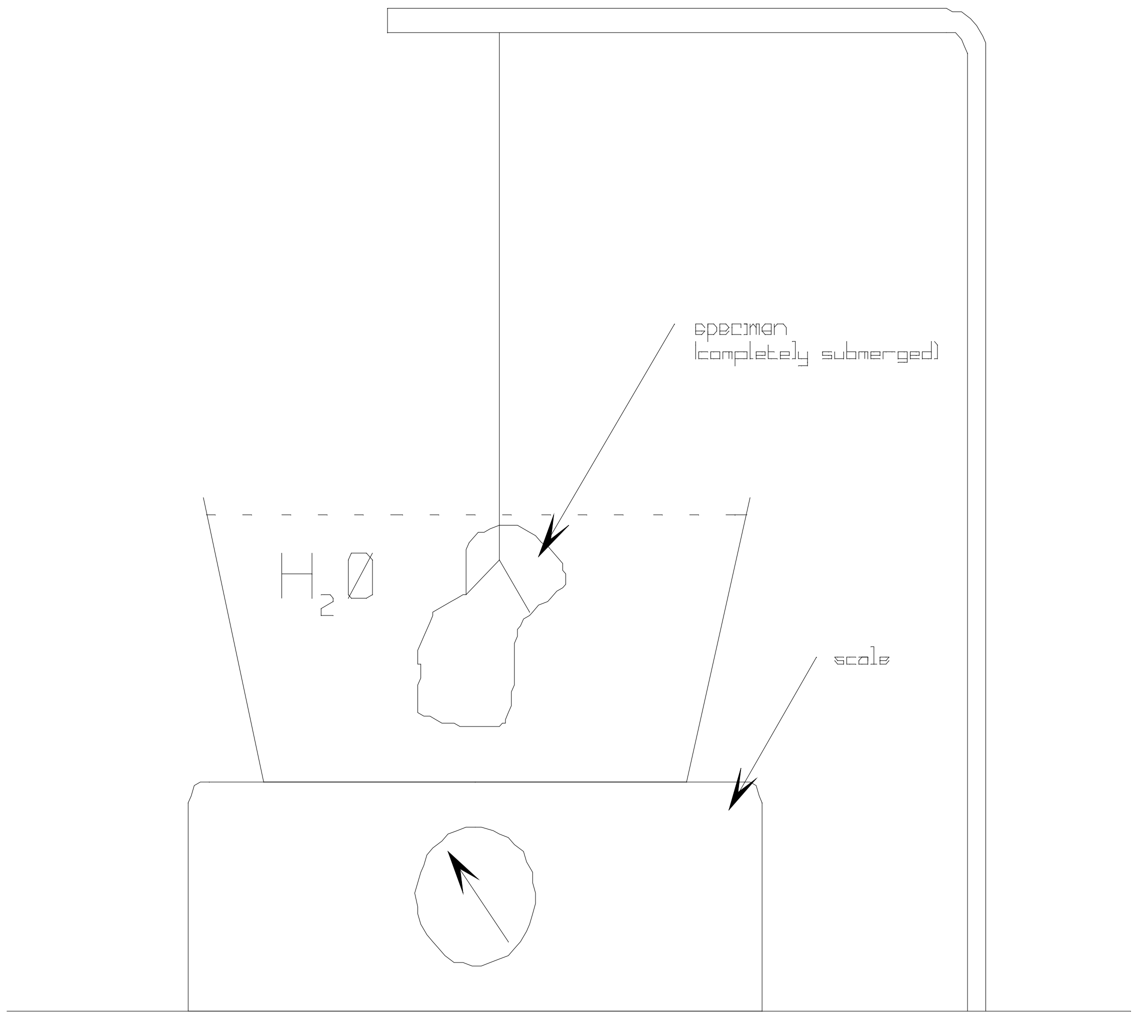
3. Poisson's constant, μ = *-transverse strain/imposed axial strain*

4. Q = *2π*(energy stored/energy lost) per cycle of vibration*

5. Maximum cyclic operating stress, σ_p

Density

- Density given by foundry is usually accurate. Density given by mill certification should be checked.
- Density of an irregular blank not having uniform dimensions can be found by buoyancy measurements.
- Measure the specimen's weight, W_s. in grams.
- Then fill a container with enough water to permit complete suspended submersion of specimen.
- Weigh the container filled with water.
- Suspend the specimen by thread whose weight is neglible compared to the specimen in the container so that is completely submerged.



specimen
(completely submerged)
H_2O
scale

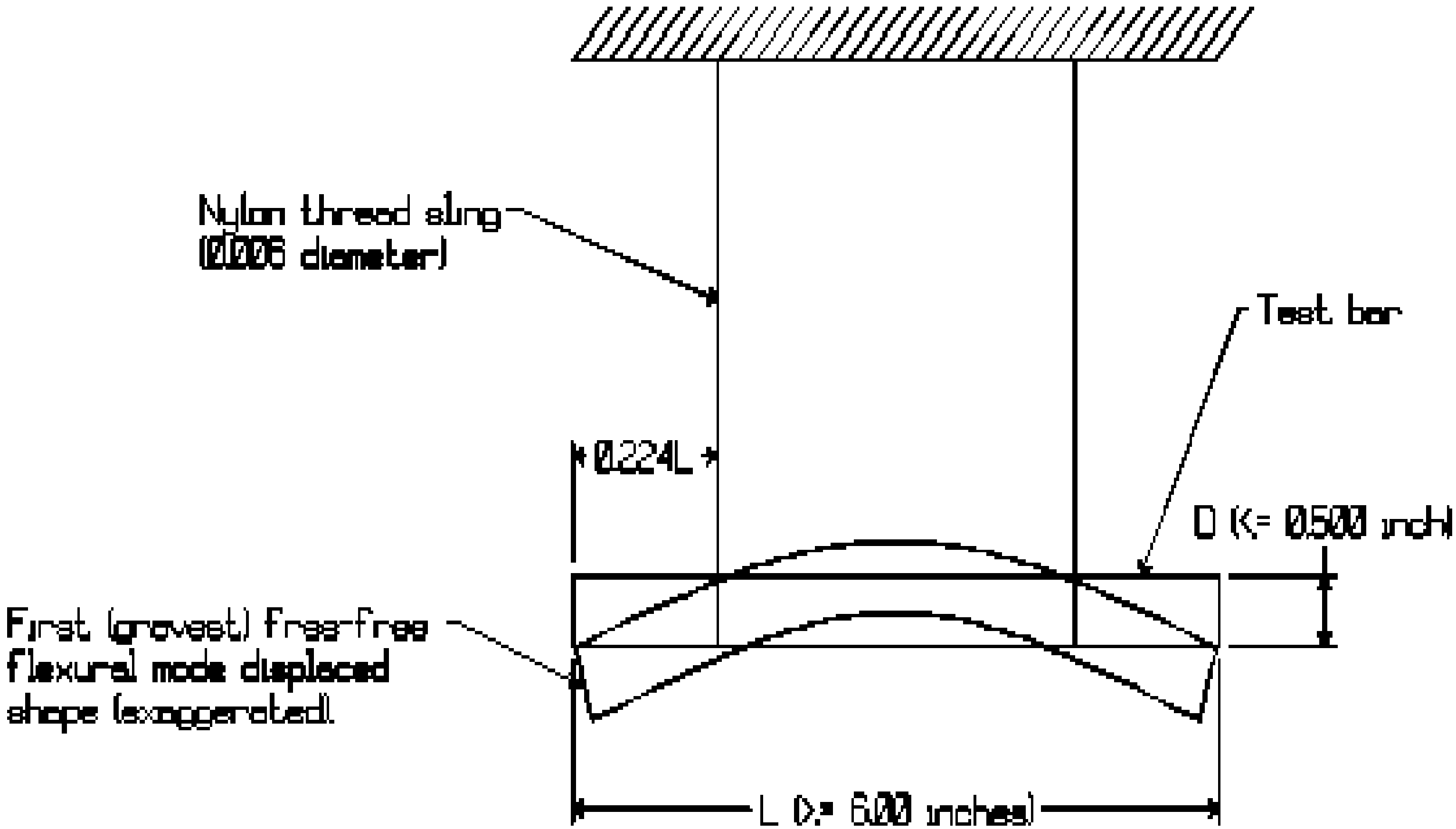
- Difference in weight measured before and after submersion, δw, = weight of volume of water displaced from which the volume of the specimen, V_s, can be found in cubic centimeters, knowing that the density of water is 1 gm/cc.: $V_s = \delta w$, cc when δw is measured in grams.

- The density of the specimen, ρ_s, is then $\rho_s = W_s/V_s$, when W_s is given in grams.

Young's modulus

Chiming Method

Reference: http://www.modalmechanics.com/UIA_1998_presentation.htm



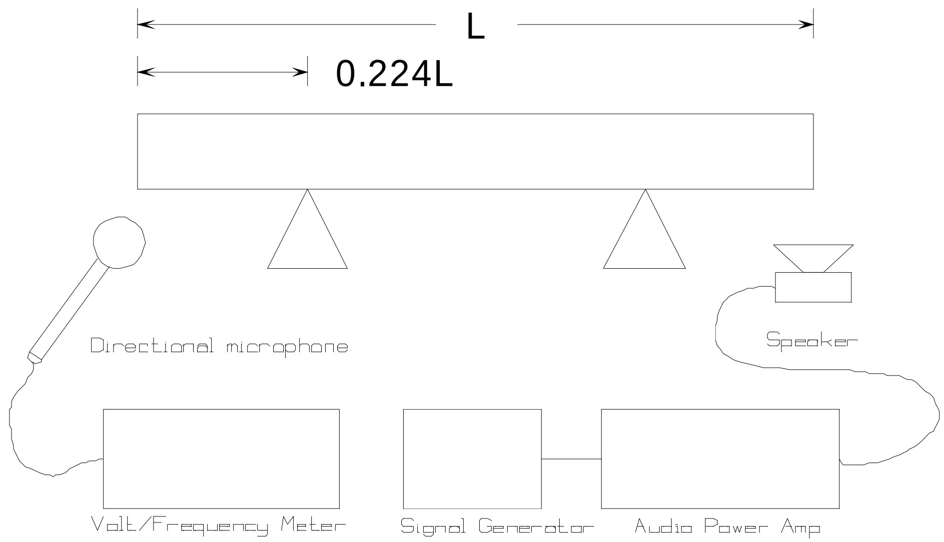
Chiming arrangement for measurement
of mechanical Q.

- A sample specimen of the material in bar form is suspended by two threads, each postioned 0.224 of the bar's length, L, from its ends.

- The bar is struck at its center with a soft faced hammer and the frequency of vibration is measured using microphone connected to a frequency meter.

- The extension sound velocity, c, of the material can be found from the equation relating the frequency of the first free-free flexural mode to the bar's dimensions: $c = 8L^2 f / \pi\kappa(3.0112)^2$ where f is the measured frequency of vibration and κ is the radius of gyration of the bar cross section with respect to the neutral axis of bending. For a round bar of diameter d, κ = d/4. For a rectangular bar of thickness t in the bending plane, $\kappa = t/\sqrt{12}$.

- Knowing c and ρ, Young's modulus, E, may be found as $E=\rho c^2$



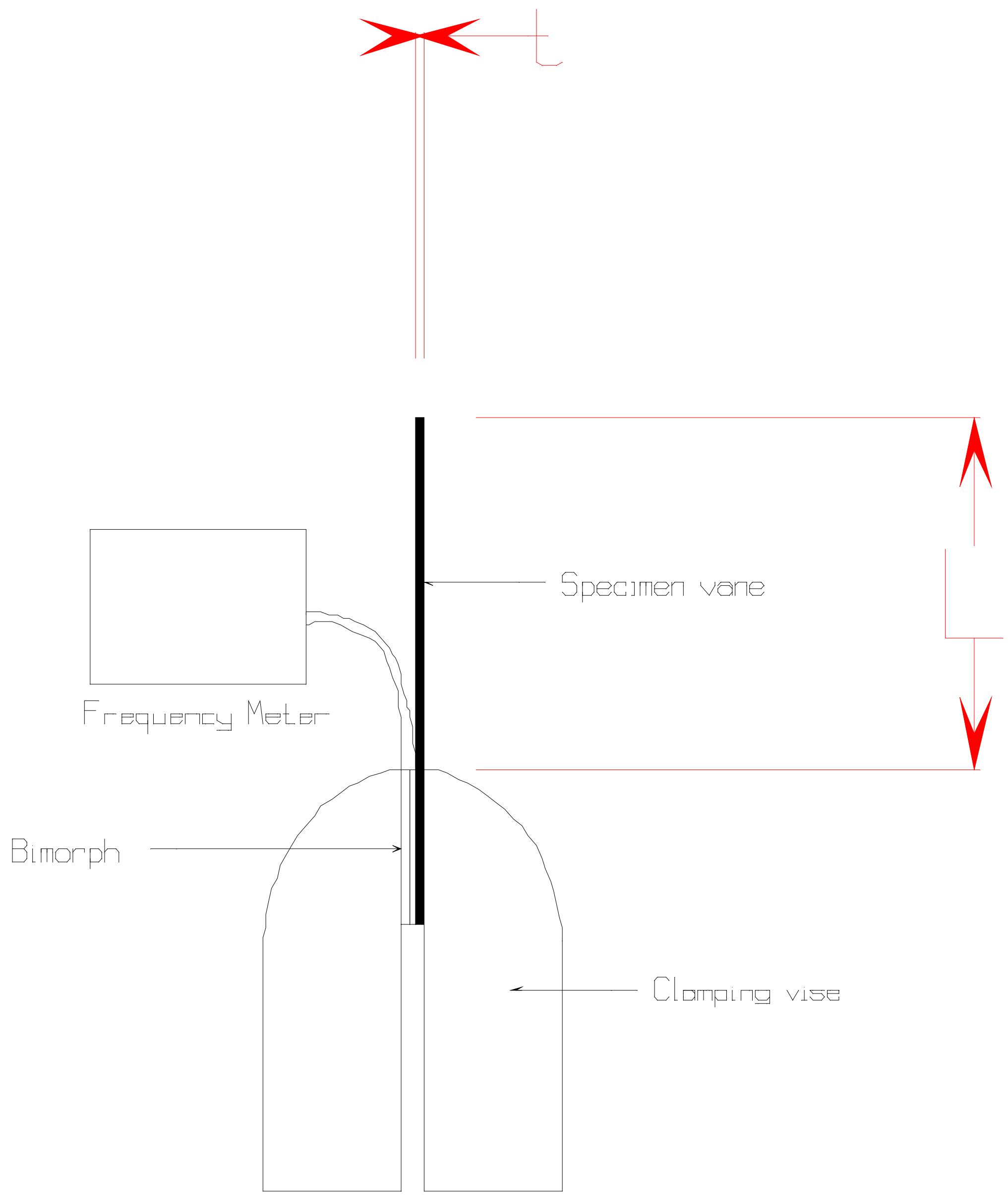
Steady state excitation method

For materials such as many thermoplastics, where elastic losses limit the duration of chiming and make frequency measurements difficult, steady state excitation of free-free flexural resonance may be used to determine c.

- Bar is supported by knife-edges or thread at each node of the first free-free flexural resonance.
- Frequency of signal generator is adjusted until signal from microphone attains a peak value.
- Knowing *f* the extensional sound velocity, and thus E, can be computed as previously described.

Fixed-Free flexural chiming method

Useful for measuring Young's modulus for metal and plastic sheet and film.


t
Specimen vane
L
Frequency Meter
Bimorph
Clamping vise

- Specimen vane is clamped between a bimorph and vise jaws.

- Bimorph electrodes are attached to a frequency meter.

- Vane is deflected and released.

- Frequency of vibration, ***f***, as read by frequency meter, is related to the extensional sound velocity as

$$c = 8fL^2 / \pi\kappa(1.194)^2$$

$$\text{where } \kappa = \mathrm{t}/\sqrt{12}$$

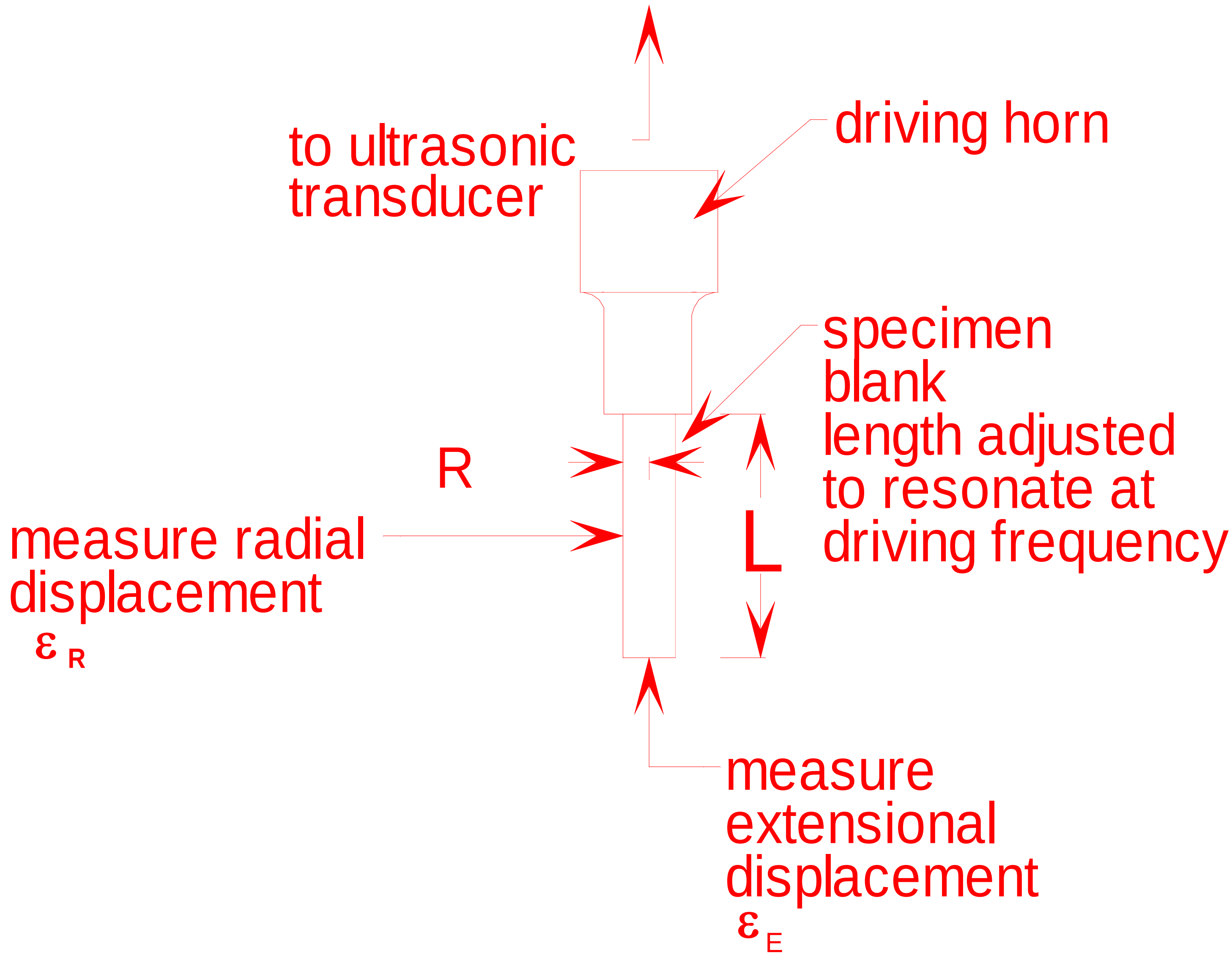
Poisson's constant

Foundries may not tabulate this constant in their listing of material properties. However an accurate assessment of its value is essential to the design of horn whose lateral dimensions are comparable to their length.

Driven resonance method

The radial displacement at the surface of the center of the specimen is μ*extensional strain at the center of specimen's axis*the radius R.

For a slender uniform half wavelength cylinder, the extensional strain at the axial center of the specimen is equal to the extensional velocity at the center of the free end divided by the extensional sound velocity, c.


to ultrasonic transducer
driving horn
specimen blank length adjusted to resonate at driving frequency
R
L
measure radial displacement
ε_R
measure extensional displacement
ε_E

- For results accurate to about two percent, machine right round specimen blank, whose length is 5 or more times its diameter, to resonate at the same frequency, ***f***, as the ultrasonic transducer+driving horn.

- For specimens whose diameter approximately equals their length, the computation of μ given below is accurate to within ten percent.

- Drive blank at a measured extension displacement, ε_E.

- Measure radial displacement , ε_R, of the specimen blank at L/2, the center of the blank length.

- $$\mu = \left(\frac{\varepsilon_R}{\varepsilon_E}\right)\left(\frac{c}{\omega R}\right)$$ where c is the extensional sound velocity, and $\omega=2\pi f$

FEA assisted measurement of Poisson's Constant

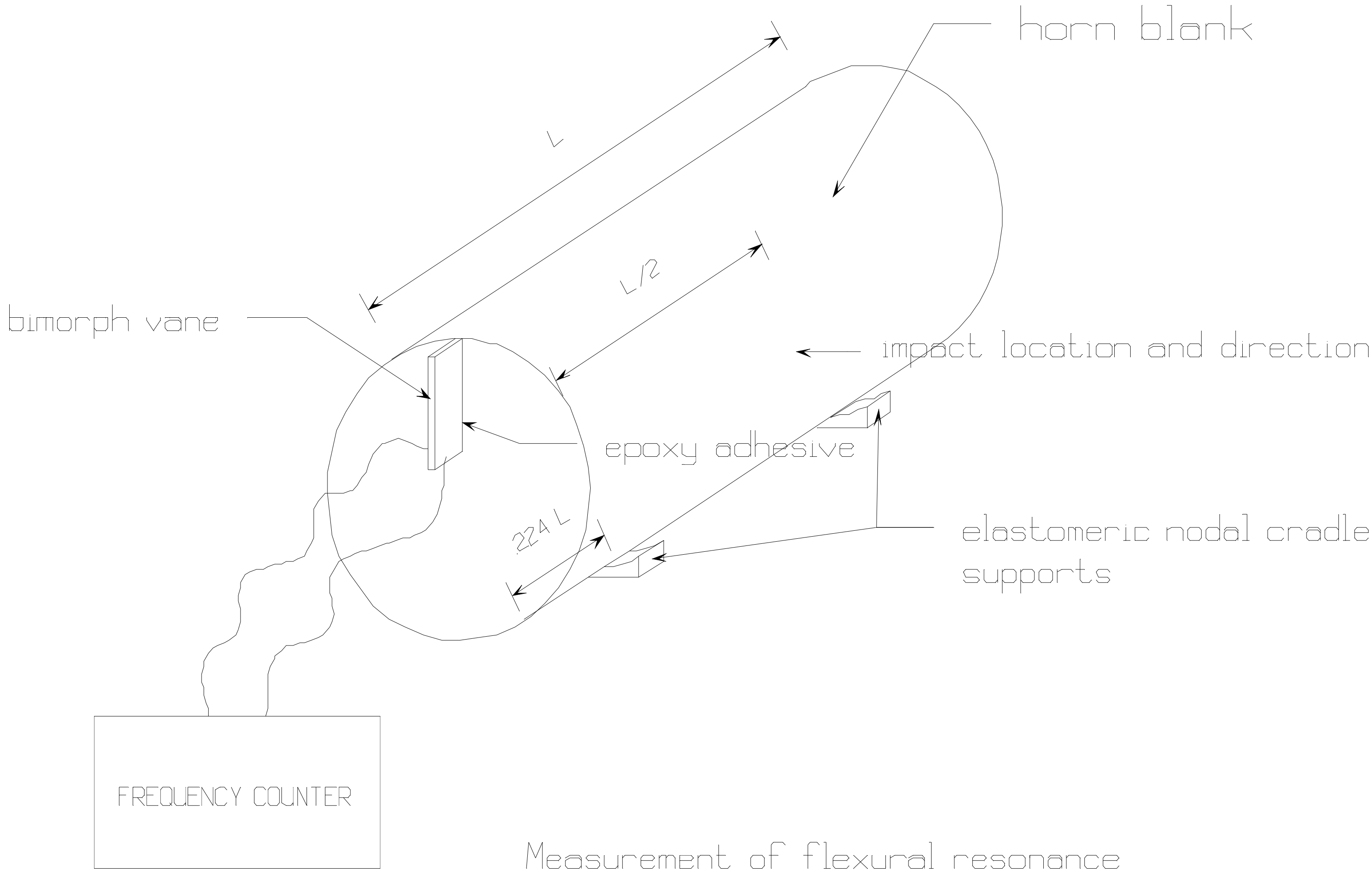
Reference:
http://www.modalmechanics.com/UIA_2002_presentation_Poisson&Young's.htm

Method takes advantage of the fact that flexural resonant vibration is far less dependent upon Poisson's constant than is torsional resonance. It also does not require a restriction on the ratio of specimen length to lateral dimension or that the specimen dimensions be adjusted to resonate with an excitation source.

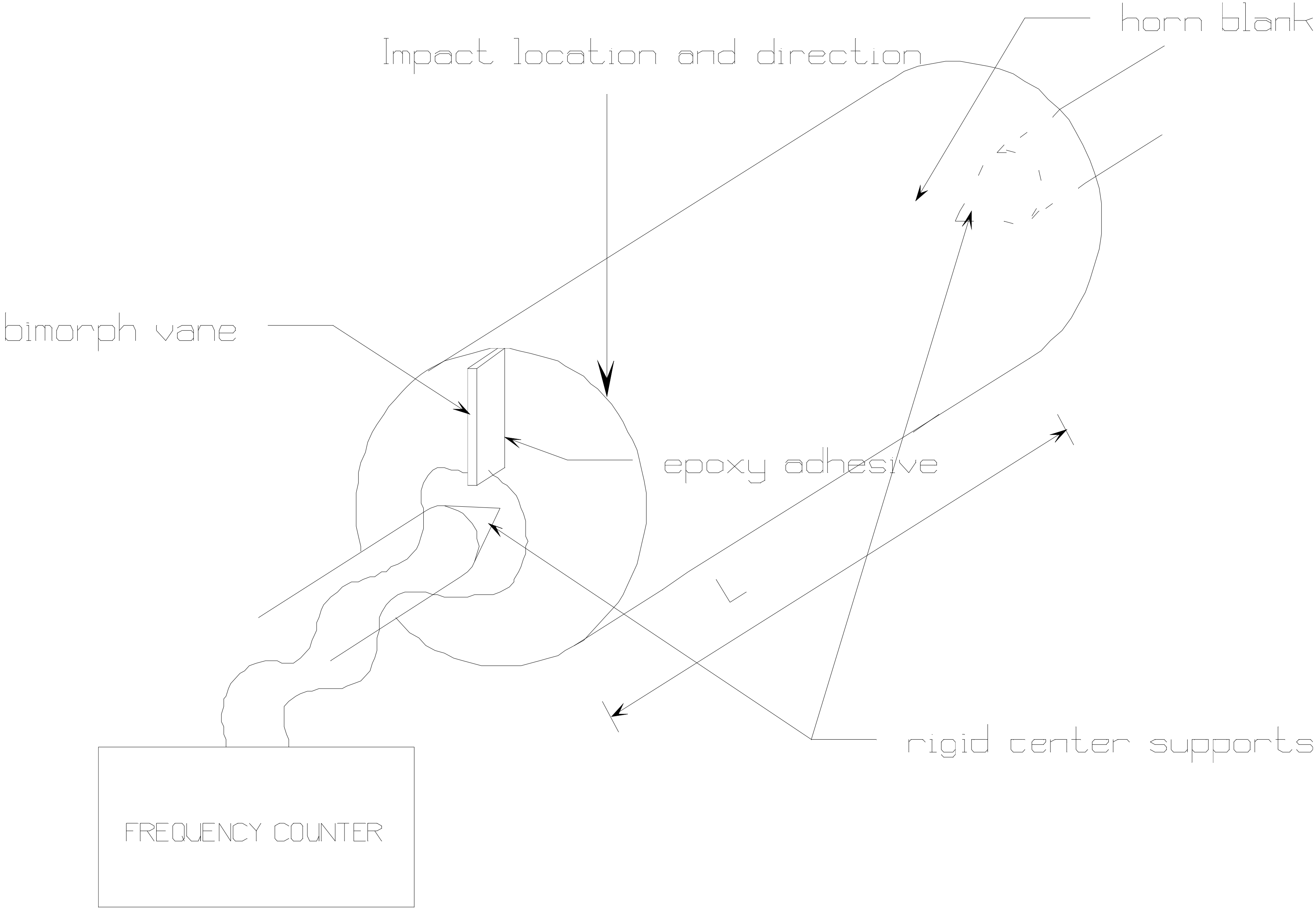
Flexural vibration frequency is proportional, in the first approximation, to the square root of Young's modulus.

Torsional vibration frequency, to the same approximation, is also dependent upon the square root of the modulus of rigidity define as $G = E/2(1+\mu)$.

Given a specimen blank that can be accurately modeled in an FEA program, the blank is excited into free-free flexural and then torsional free-free vibration as shown below and the frequencies measured. Determination of the correct mode frequency range can usually be determined from a preliminary FEA modal analysis using nominal values for E and μ.



Measurement of flexural resonance


horn blank
Impact location and direction
bimorph vane
epoxy adhesive
L
rigid center supports
FREQUENCY COUNTER

- Using a nominal value for both E and μ, the frequency of the first free-free flexural resonance is computed from FEA modal analysis.

- The value of E is then adjusted by multiplying the initial value of E by the square of the ratio of computed flexural resonant frequency measured to that measured.

- $$E_2 = (\frac{fc}{fm})^2 E$$

- where f_c and f_m are the computed and actual measured flexural resonant frequencies.

- The value E_2 is then assigned to the material and a second modal analysis performed, resulting in a flexural resonance close to that measured and also resulting in a new computed torsional frequency, f_{t2}.

- The value of μ is then adjusted by multiplying the quantity (1+μ) by the square of ratio of measured to computed frequency and subtracting 1 from this result:

- $$\mu_2 = (1+\mu)\left(\frac{f_{tm}}{f_{t2}}\right)^2 - 1$$

- where f_{tm} and f_{t2} are the measured and computed torsional frequencies. Note that frequency ratio is, for this correction to μ, the inverse of that for E as the square of the torsional frequency is inversely proportional to Poisson's constant.

- Modal analysis is again performed, resulting in new computed flexural and torsional resonant frequencies. Again the value of E is adjusted to E_3 by multiplying the value of E_2 by the square of the ratio of the measured to the computed frequency.

- Modal analysis is again performed to adjust μ as described above.

- The iterations continue until the desired level of agreement between the measured flexural and torsional resonances with the computed values is achieved. Typically, three iterations are required to achieve an accuracy of better than 99 percent.

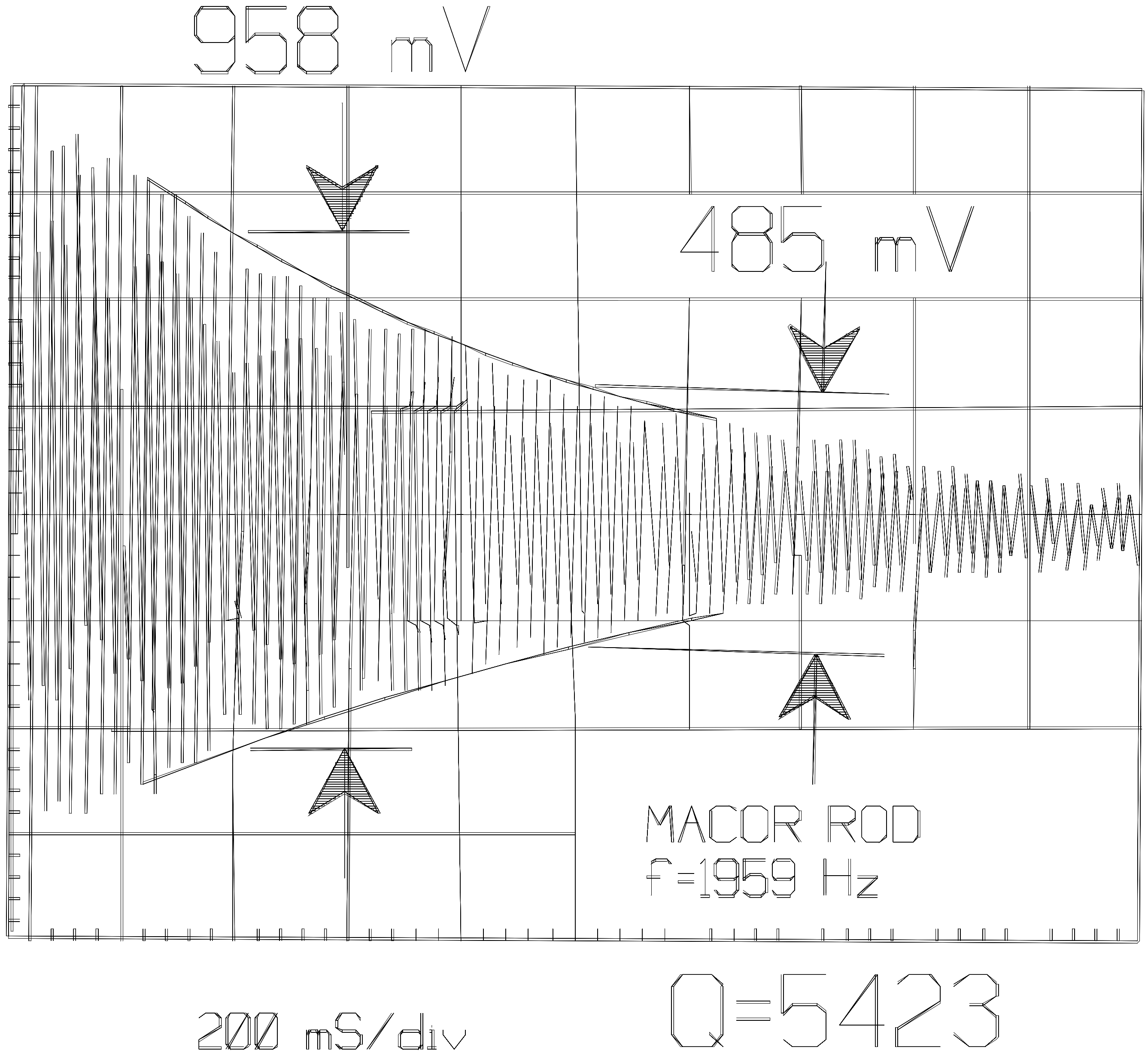
Q and Cyclic Fatigue

Reference: http://www.modalmechanics.com/UIA_1998_presentation.htm

The mechanical Q of a horn determines its power consumption at any given level of vibration and thus its temperature rise and, ultimately, its ability of withstand vibration without failure as fatigue strength varies inversely with temperature.

For low levels of cyclic stress, chiming, previously described in the measurement of E, affords a simple method of measurement.

From measurement of the decay of vibration amplitude or, as it sometimes termed, ring-down, as recorded on an oscilloscope depicting the microphone signal, Q can be rapidly determined as shown below for Macor ceramic bar chime.


958 mV
485 mV
MACOR ROD
f=1959 Hz
200 mS/div
Q=5423

The Q can be found by remembering that the Q is defined as 2π times the ratio of total vibrational energy to the energy lost per cycle and that vibration energy is proportional to the square of the amplitude, it can be shown, since the rate at which energy is lost per cycle is proportional to the total energy of vibration at any time, that the amplitude, ξ, at any point on a resonator initially excited into resonant vibration, and then allowed to freely decay, must diminish exponentially:

$$\xi / \xi_o = \exp\left(-\frac{\pi f t}{Q}\right),$$

where f is the frequency, t is the time of the observation and ξ_o is the initial amplitude.

If amplitudes ξ_1 and ξ_2 are measured at times t_1 and t_2, from the above equation ,

$$Q = \frac{\pi f \Delta t}{\ln\left(\xi_1 / \xi_2 \right)}$$

where $t_1 - t_2 = \Delta t$.

Measuring the amplitude of the decay, ξ_1 , at time t_1 and then again ξ_2 at time t_2, and knowing the frequency f, the Q may be computed. The table below gives the value of Q measured using this method on a variety of materials.

Figure 1

Magnetically levitate d test chime

Material	Condition	Q
6Al-4V Titanium	Lots, as received	2,000-6,000
6Al-4V	Annealed	18,000-22,000
17-4PH Stainless Steel	Annealed	7,000
PH15-7Mo Stainless Steel	Annealed	17,000
PH15-7Mo	Hardened	17,000
PH13-8Mo Stainless Steel	Annealed	10,000
Custom 455 Stainless Steel	H900 condition	10,000
MACOR^{TM}	As received	5,000

Chiming Q of Various Materials

However, caution is advised in using these values to predict operation at large strains as only in 6Al-4V is the Q invariant with cyclic strain until a level of about .25 percent is reached. Other metals, particularly the 300 series stainless steels and precipitation hardened stainless steels, such as 17-4 PH, exhibit a declining value of Q with increased cyclic strain.

Other researchers have reported values of Q at various strains for a variety of materials, as shown below.

Figure 1

Measured room temperature Quality factors for selected materials

Material	Q	Notes
90Ti-6Al-4V Titanium alloy	20,000	Annealed. Strain< 0.003, 17 kHz
Low carbon steel	250	Annealed, Strain <0.0001, 23 kHz
Lead	500	Commercially pure,Strain < 0.00025
Aluminum	10,000	
Magnesium	5700	
Tungsten carbide steel	8180	
Bakelite	200	Strain < .0024, 17.6 kHz
Polycarbonate (unreinforced)	100	Strain 0.005, 20kHz
Unfilled polypropylene	100	Strain 0.005, 20 kHz

Measurement of Q at large strains by power consumption

At appreciable strains, the Q may be measured by machining a wavelength rod of the material resonant at the frequency of the driving transducer+horn and measuring the electrical power, P_1, consumed by the transducer when driving this rod. The rod is then shortened by a half wavelength and the power consumed in driving the rod, P_2, is measured.

The energy stored in a half wavelength prismatic rod is equal to one quarter of the mass of the rod, M, multiplied by the free end velocity, V_o, squared:

$$E=m(V_o)^2/4$$

The energy consumed per cycle of vibration, E_d, is the power consumption measured divided by the frequency of vibration:

$$E_d=P/f$$

The difference P_1-P_2 is the power consumed by a half wavelength of the material, hence:

$$Q=2\pi[\text{energy stored per cycle/energy lost per cycle}] =$$

$$\pi\,[m(V_o)^2 f]/[2(P_1-P_2)],$$

where $V_o = 2\pi f\xi$, ξ being the observed free end displacement of the rod which is kept the same for both P_1 and P_2 measurements.

This method takes into account any losses produced the coupling of the rod to the driving horn, but it does not take into account the energy conversion efficiency of the transducer. Typically a piezo-electric transducer's efficiency is in the range of 95 percent. Measurements using this technique are given by

http://www.modalmechanics.com/UIA_2002_presentation_Q_stainless_steels.htm

If greater accuracy is desired, interested parties are referred to a method for measuring Q that relies wholly upon the temperature rise of a specimen that is given in

http://www.modalmechanics.com/Acoustic_loss_at_substantial_ultrasonic_strain_in_6Al-6V-2Sn_and_sintered_6Al-4V_Titanium.htm

The table below summarizes some of the results obtained by thermal measurements.

sampleD	Diameter x length mm	Density kg/m^3	Young's Modulus Gpa	cyclic strain percent	cyclic stress Mpa (kpsi)	Q	comments
Custom 455	12.6 x 126	7750	200	0.06	124 (18)	1800	warm at center
				0.09	250 (24)	1800	hot at center
Aeromet 100	19.3 x 123	7789	194	0.06	124 (18)	5900	warm at center
				0..09	250 (24)		fractured
6Al-4v Titanium (heat treated)	9.3 x 127	4429	110	0.06	70 (10)	2800	
				0.09	100 (15)	2800	
				0.12	140 (20)	2700	warm at center

FEA assisted prediction of expected horn losses at specified operating levels.

The expected power loss in a horn can be estimated if the Q of the material is known for the maximum operating stress.

The method relies upon the definition of resonance as the cyclic exchange of kinetic and potential energy. Kinetic energy is equal to the sum of each mass element multiplied by half its peak velocity squared. The potential energy, E_P, is equal to the half the sum of the stress in each mass element squared divided by Young's modulus and is equal at resonance to the kinetic energy, E_k:

$$E_K = \Sigma_i (m_i v_i^{\,2})/2 = \omega^2 \Sigma_i (m_i \xi_i^{\,2})/2$$

where $v = \omega\xi$, ξ being the displacement of the mass element.

$\omega^2 = 2E_K / \Sigma_i (m_i \xi_i^{\ 2}) = 2E_P / \Sigma_i (m_i \xi_i^{\ 2})$, as, at resonance $E_K = E_{P.}$

Now suppose material is removed from the horn where there is little motion but substantial stress, such as the regions about a motional node. The potential energy thus is decreased but, if the vibration amplitude is kept the same, the sum $\Sigma_i(m_i \xi^2_i)/2$ does not change. Then

$$\omega_1^{\ 2} = 2E_{P1} / \Sigma_i (m_i \xi_i^{\ 2}) = E_{p1} / [E_{K0} / \omega_o^{\ 2}]$$

where the subscripts 1 and 0 denote respectively the new resonant frequency resulting from the removal of material and the original resonant frequency.

We then have

$$(\omega_1 / \omega_0)^2 = E_{P1} / E_{K0} = (E_{P0} - \Delta E_p) / E_{K0} = 1 - \Delta E_p / E_K$$

Modal analysis provides both the resonant frequency, the amplitude and stress distributions in the unaltered horn permitting calculation of the potential energy in a small amount of mass undergoing little displacement in comparison to other parts of the horn and that is removed in the altered horn. The analysis is then repeated with this mass removed and a new resonant frequency computed. Since then both and ω_0 and ω_1 as well as ΔK_p are known, E_{K0} can be found as:

$$E_{K0} = \Delta E_P / [1 - (\omega_1 / \omega_o)^2]$$

Having found the stored energy of vibration at a particular level of stress, the power dissipation ,P, in the horn follows, from the definition of Q, as

$P = f(\text{energy lost per cycle}) = 2\pi f E_{K0}/Q$.

In a like manner, if some mass is added or subtracted from a horn where there is maximum motion and neglible stress, the stored energy of vibration can be found computing the kinetic energy, ΔK_e, of the mass removed from the modal analysis results for the unaltered horn as:

$$K_{E0} = \Delta K_e / [1 - (\omega_0 / \omega_1)^2] .$$

Note that this analysis assumes that the Q is independent of stress. Using the value of Q appropriate to the largest value of stress in the horn will then provide an upper limit to power consumption.

Maximum safe operating stresses (strains) in ultrasonic horns.

Reference:
http://www.modalmechanics.com/Acoustic_loss_at_substantial_ultrasonic_strain_in_6Al-6V-2Sn_and_sintered_6Al-4V_Titanium.htm

It has generally been determined, from stress versus cycles to failure data obtained from applying repetitive cyclic stress to specimens, that indefinite life in vibration can be obtained ***in metals*** by choosing the maximum operating stress to be no more than one third of the yield stress for the material. For example, the yield stress of 6Al-4V titanium is about 120,000 psi (827 MPa). Indefinite horn life can be achieved by ensuring that no part of the horn experiences more than 40,000 psi (275 MPa).

It is possible to also design plastic horns, as has been demonstrated by the technically, if not commercially, successful development of the ultrasonic tooth brush. For a material such as polycarbonate, the yield stress is about 9,000 psi (62 MPa). However, it has been found that above a 20 kHz cyclic stress of about 400 psi (2.7 MPa), which corresponds to a free end peak-peak displacement of a simple half wavelength rod of 0.6 mils (15 μ), failure occurs due to softening of the material caused by the elevation in temperature from cyclic stress loss. The Q of polycarbonate has been measured to be about 70! Nevertheless, its tested use as low frequency resonators, such as those operating in 50-150 Hz region where the power loss is reduced by low frequency operation, has shown that it may operate indefinitely providing the maximum cyclic stress is kept below 2,000 psi (1.4 MPa). The table below provides data on the low frequency cyclic endurance of some common thermoplastics.

Thermoplastic film	Thickness (inch)	Condition	Density (lbsf/in^3)	Modulus (psi)	Q
PC (polycarbonate)	0.022	annealed 250F 1 hour	0.04	260,000	70
PVC (polyvinylchloride)	0.015	cut from packaging	0.048	460,000	55
PET (polyethylene terathylate)	0.012	cut from packaging	0.047	860,000	100
PS (polystyrene)	0.013	cut from packaging	0.038	570,000	100

Thermoplastic film	Indefinite cyclic life fatique limit (psi)	Notes
PC (polycarbonate)	2,000	74 Hz, vane crack at 2300 psi
PVC (polyvinylchloride)	2,000	63 Hz, vane crack at 2500 psi
PET (polyethylene terathylate)	>4,500	93 Hz, cyanoacrylate adhesive failure, vane intact
PS (polystyrene)	3,500	99 Hz, vane crack at 3750 psi

Table 1

Effect of residual stress upon Q

Material for horns is commonly obtained in round rod or billet form. Stress induced during rolling or rerolling or by forging have been shown to substantially affect the Q and thus running power consumption. For 6Al-4V Titanium, chiming measurements of blanks and tests made upon horns finished from blanks showed that tips operating at 40,000 psi (275 MPa) peak stress made from blanks having a Q below about 7500 heated sufficiently to cause tensile failure.

Residual stress may be removed by annealing material prior to machining. In 6-4 Ti, annealing will, in general, restore the Q to its established level of about 20,000. Annealing of other Titanium alloys as well as the stainless steels is also recommended to obtain optimum performance.

Heat treatment to increase the yield strength of precipitation hardenable stainless steels does not produce the expected increase in free face displacement as many of these treatments, especially those for 15-7PH ,17-4 PH, Aeromet 100 and Custom 455, lower the Q and cause failure from an elevation in temperature during operation. The Q for all steels measured decreases, in any case, with strain. In designing stainless steel horns, it is best to choose materials with the highest yield strength in the annealed state and to disregard the higher strength available from heat treatment.

Fini

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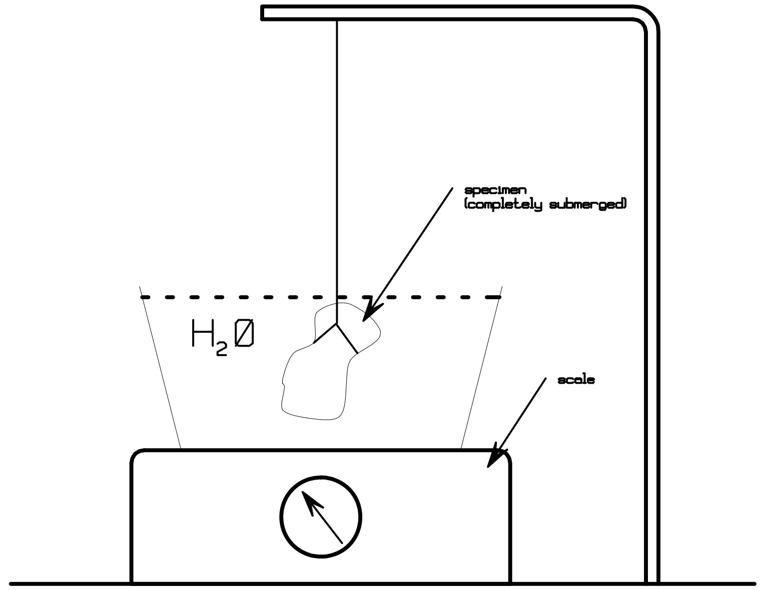

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Young's modulus

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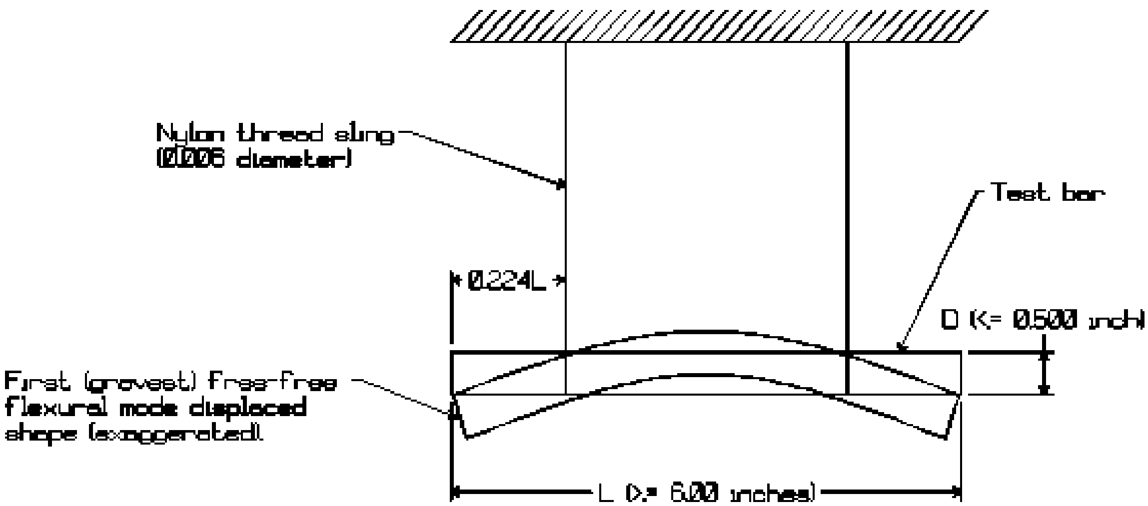

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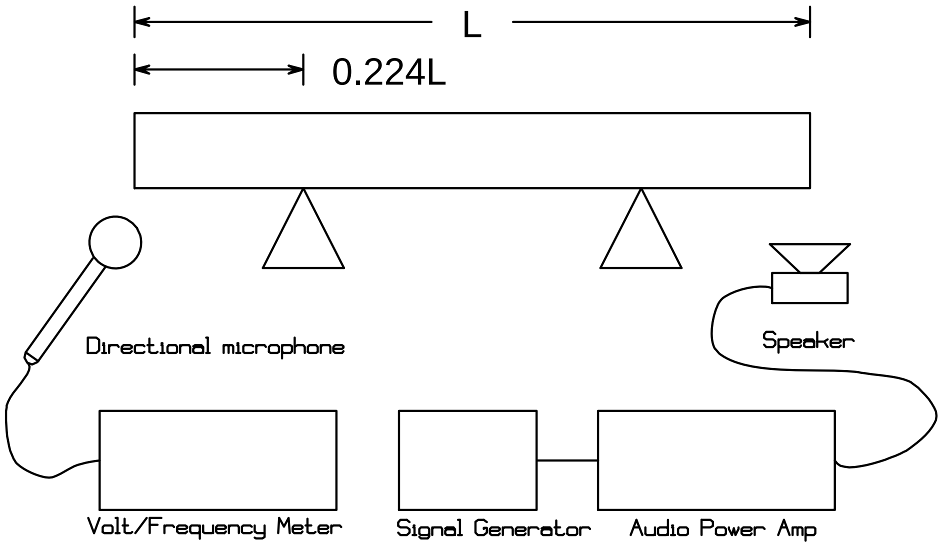

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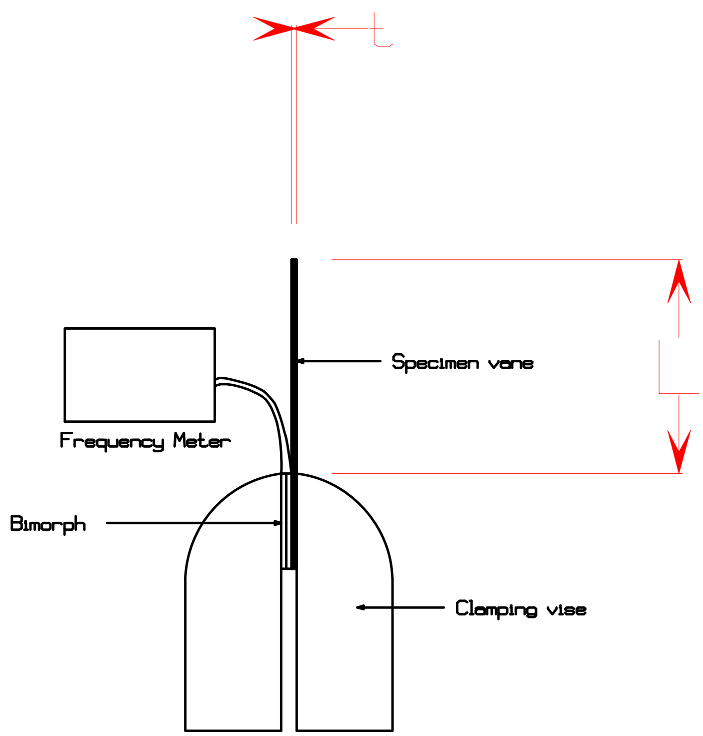

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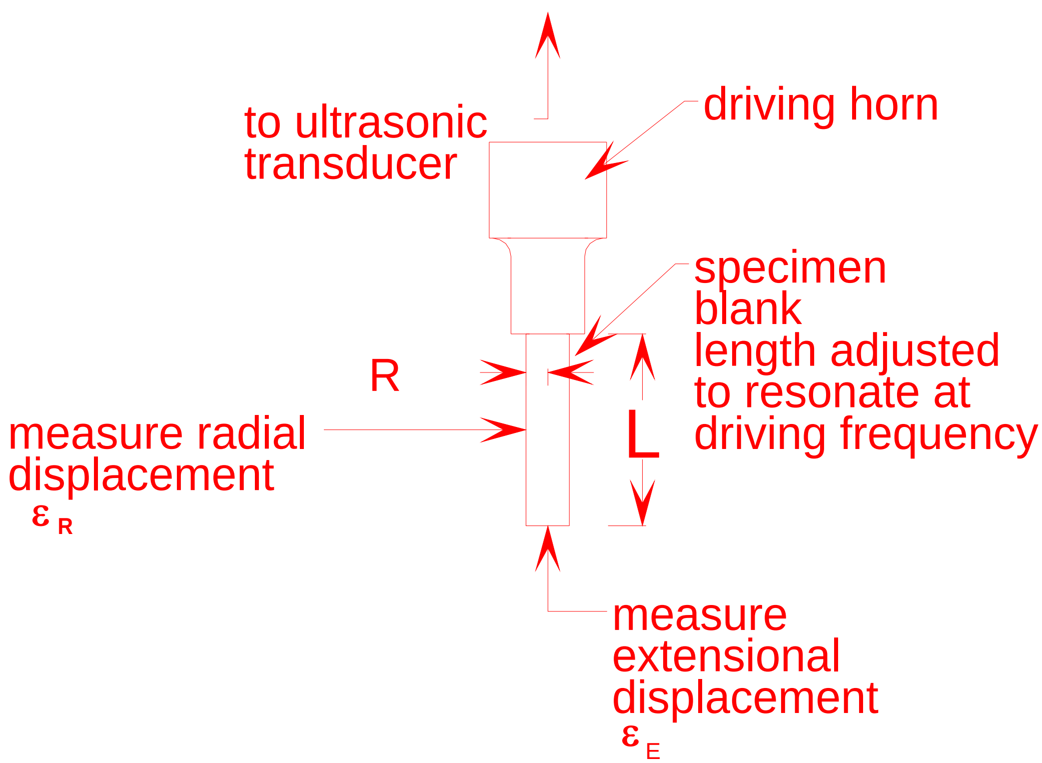

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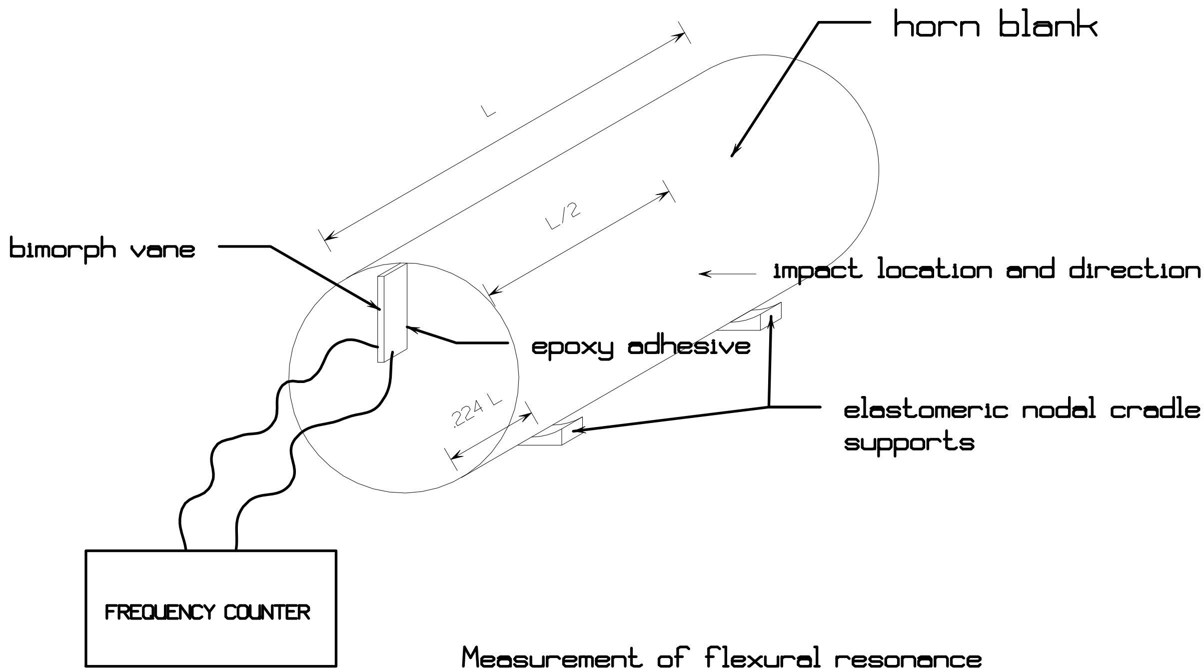
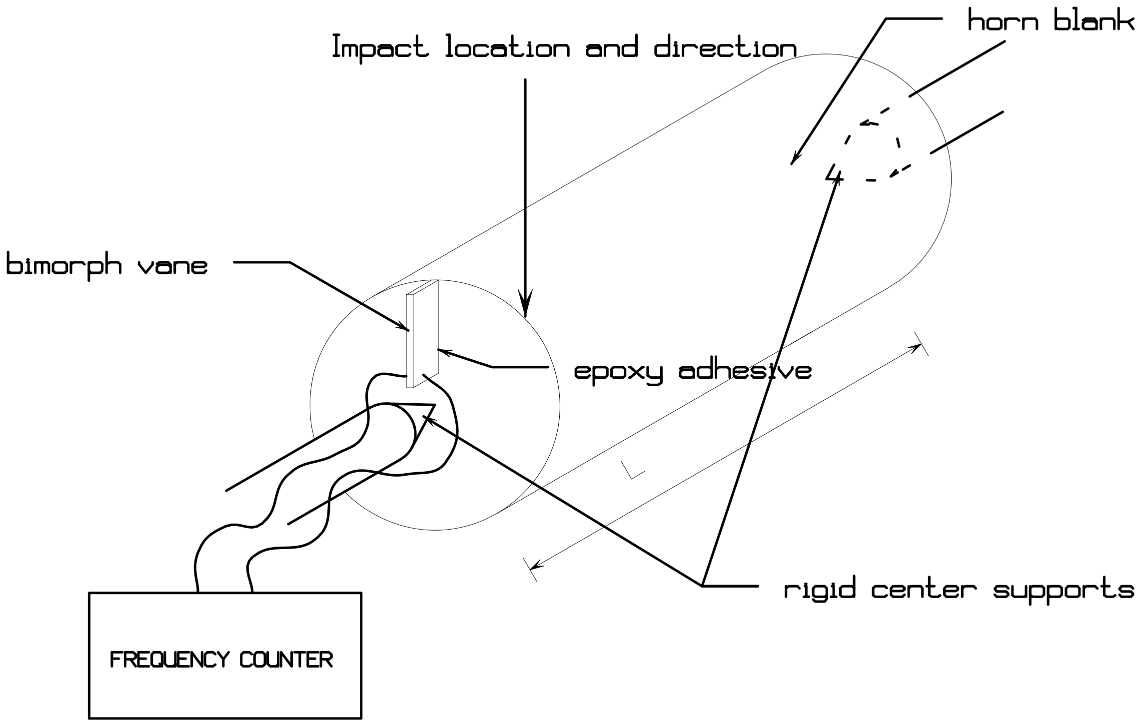


Measurement of flexural resonance



Measurement of torsional resonant frequency

- Using a nominal value for both E and μ, the frequency of the first free-free flexural resonance is computed from FEA modal analysis.

- The value of E is then adjusted by multiplying the initial value of E by the square of the ratio of computed flexural resonant frequency measured to that measured.

- $$E_2 = (\frac{fc}{fm})^2 E$$

- where f_c and f_m are the computed and actual measured flexural resonant frequencies.

- The value E_2 is then assigned to the material and a second modal analysis performed, resulting in a flexural resonance close to that measured and also resulting in a new computed torsional frequency, f_{t2}.

- The value of μ is then adjusted by multiplying the quantity (1+μ) by the square of ratio of measured to computed frequency and subtracting 1 from this result:

- $$\mu_2 = (1+\mu)(\frac{f_{tm}}{f_{t2}})^2 - 1$$

- where f_{tm} and f_{t2} are the measured and computed torsional frequencies. Note that frequency ratio is, for this correction to μ, the inverse of that for E as the square of the torsional frequency is inversely proportional to Poisson's constant.

- Modal analysis is again performed, resulting in new computed flexural and torsional resonant frequencies. Again the value of E is adjusted to E_3 by multiplying the value of E_2 by the square of the ratio of the measured to the computed frequency.

- Modal analysis is again performed to adjust μ as described above.

- The iterations continue until the desired level of agreement between the measured flexural and torsional resonances with the computed values is achieved. Typically, three iterations are required to achieve an accuracy of better than 99 percent.

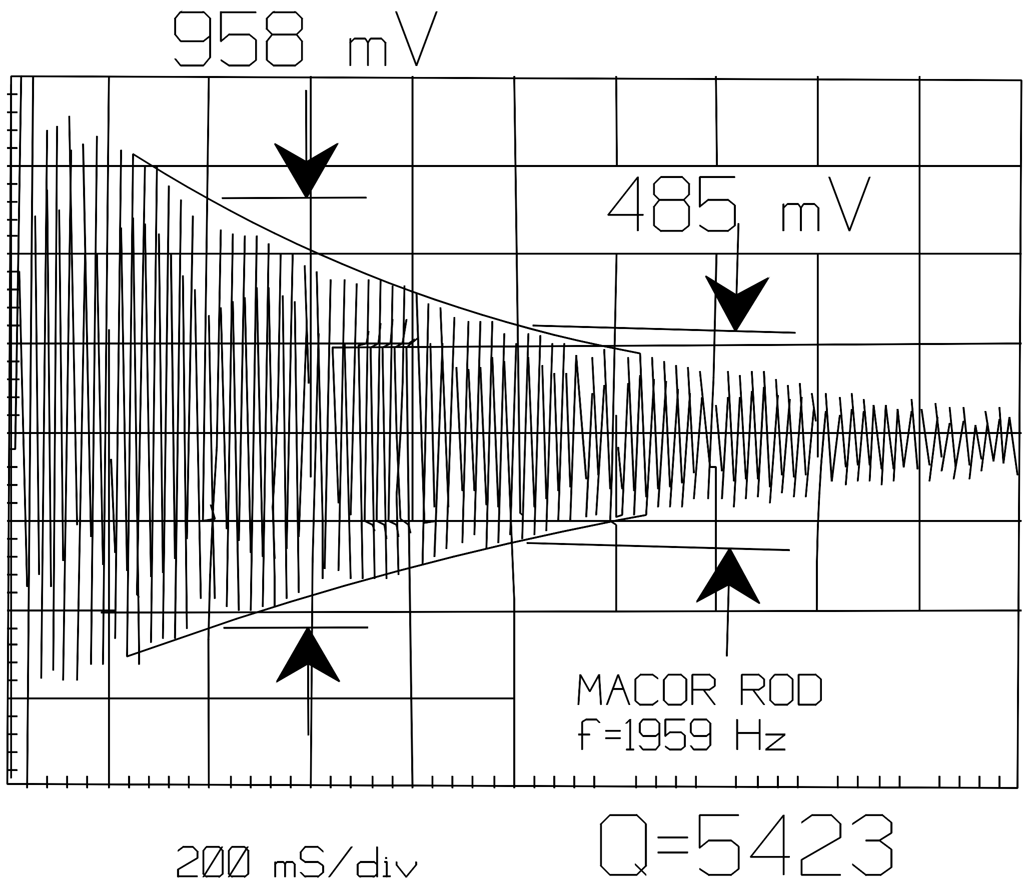
Q and Cyclic Fatigue

Reference: http://www.modalmechanics.com/UIA_1998_presentation.htm

The mechanical Q of a horn determines its power consumption at any given level of vibration and thus its temperature rise and, ultimately, its ability of withstand vibration without failure as fatigue strength varies inversely with temperature.

For low levels of cyclic stress, chiming, previously described in the measurement of E, affords a simple method of measurement.

From measurement of the decay of vibration amplitude or, as it sometimes termed, ring-down, as recorded on an oscilloscope depicting the microphone signal, Q can be rapidly determined as shown below for Macor ceramic bar chime.



The Q can be found by remembering that the Q is defined as 2π times the ratio of total vibrational energy to the energy lost per cycle and that vibration energy is proportional to the square of the amplitude, it can be shown, since the rate at which energy is lost per cycle is proportional to the total energy of vibration at any time, that the amplitude, ξ, at any point on a resonator initially excited into resonant vibration, and then allowed to freely decay, must diminish exponentially:

$$\xi / \xi_o = \exp\left(-\frac{\pi f t}{Q}\right),$$

where *f* is the frequency, *t* is the time of the observation and

ξ_o is the initial amplitude.

If amplitudes ξ_1 and ξ_2 are measured at times t_1 and t_2, from the above equation ,

$$Q = \frac{\pi f \Delta t}{\ln\left(\xi_1 / \xi_2 \right)}$$

where $t_1 - t_2 = \Delta t$.

Measuring the amplitude of the decay, ξ_1 , at time t_1 and then again ξ_2 at time t_2, and knowing the frequency f, the Q may be computed. The table below gives the value of Q measured using this method on a variety of materials.

Figure 1

Magnetically levitated test chime

Material	Condition	Q
6Al-4V Titanium	Lots, as received	2,000-6,000
6Al-4V	Annealed	18,000-22,000
17-4PH Stainless Steel	Annealed	7,000
PH15-7Mo Stainless Steel	Annealed	17,000
PH15-7Mo	Hardened	17,000
PH13-8Mo Stainless Steel	Annealed	10,000
Custom 455 Stainless Steel	H900 condition	10,000
MACOR™	As received	5,000

Chiming Q of Various Materials

However, caution is advised in using these values to predict operation at large strains as only in 6Al-4V is the Q invariant with cyclic strain until a level of about .25 percent is reached. Other metals, particularly the 300 series stainless steels and precipitation hardened stainless steels, such as 17-4 PH, exhibit a declining value of Q with increased cyclic strain.

Other researchers have reported values of Q at various strains for a variety of materials, as shown below.

Figure 2

Measured room temperature Quality factors for selected materials

Material	Q	Notes
90Ti-6Al-4V Titanium alloy	20,000	Annealed. Strain < 0.003, 17 kHz
Low carbon steel	250	Annealed, Strain <0.0001, 23 kHz
Lead	500	Commercially pure,Strain < 0.00025
Aluminum	10,000	
Magnesium	5700	
Tungsten carbide steel	8180	
Bakelite	200	Strain < .0024, 17.6 kHz
Polycarbonate (unreinforced)	100	Strain 0.005, 20kHz
Unfilled polypropylene	100	Strain 0.005, 20 kHz

Measurement of Q at large strains by power consumption

At appreciable strains, the Q may be measured by machining a wavelength rod of the material resonant at the frequency of the driving transducer+horn and measuring the electrical power, P_1, consumed by the transducer when driving this rod. The rod is then shortened by a half wavelength and the power consumed in driving the rod, P_2, is measured.

The energy stored in a half wavelength prismatic rod is equal to one quarter of the mass of the rod, M, multiplied by the free end velocity, V_o, squared:

$$E=m(V_o)^2/4$$

The energy consumed per cycle of vibration, E_d, is the power consumption measured divided by the frequency of vibration:

$$E_d=P/f$$

The difference P_1-P_2 is the power consumed by a half wavelength of the material, hence:

$$Q=2\pi[\text{energy stored per cycle/energy lost per cycle}] =$$

$$\pi\,[m(V_o)^2 f]/[2(P_1-P_2)],$$

where $V_o = 2\pi f\xi$, ξ being the observed free end displacement of the rod which is kept the same for both P_1 and P_2 measurements.

This method takes into account any losses produced the coupling of the rod to the driving horn, but it does not take into account the energy conversion efficiency of the transducer. Typically a piezo-electric transducer's efficiency is in the range of 95 percent. Measurements using this technique are given by

http://www.modalmechanics.com/UIA_2002_presentation_Q_stainless_steels.htm

If greater accuracy is desired, interested parties are referred to a method for measuring Q that relies wholly upon the temperature rise of a specimen that is given in

http://www.modalmechanics.com/Acoustic_loss_at_substantial_ultrasonic_strain_in_6Al-6V-2Sn_and_sintered_6Al-4V_Titanium.htm

The table below summarizes some of the results obtained by thermal measurements.

20 kHz Acoustic Loss of Heat Treated High Strength alloys

sampleD	Diameter x length mm	Density kg/m^3	Young's Modulus Gpa	cyclic strain percent	cyclic stress Mpa (kpsi)	Q	comments
Custom 455	12.6 x 126	7750	200	0.06	124 (18)	1800	warm at center
				0.09	250 (24)	1800	hot at center
Aeromet 100	19.3 x 123	7789	194	0.06	124 (18)	5900	warm at center
				0..09	250 (24)		fractured
6Al-4v Titanium (heat treated)	9.3 x 127	4429	110	0.06	70 (10)	2800	
				0.09	100 (15)	2800	
				0.12	140 (20)	2700	warm at center

FEA assisted prediction of expected horn losses at specified operating levels.

The expected power loss in a horn can be estimated if the Q of the material is known for the maximum operating stress.

The method relies upon the definition of resonance as the cyclic exchange of kinetic and potential energy. Kinetic energy is equal to the sum of each mass element multiplied by half its peak velocity squared. The potential energy, E_P, is equal to the half the sum of the stress in each mass element squared divided by Young's modulus and is equal at resonance to the kinetic energy, E_k:

$$E_K = \Sigma_i (m_i v_i^{\ 2})/2 = \omega^2 \Sigma_i (m_i \xi_i^{\ 2})/2$$

where $v = \omega\xi$, ξ being the displacement of the mass element.

$\omega^2 = 2E_K / \Sigma_i (m_i \xi_i^{\ 2}) = 2E_P / \Sigma_i (m_i \xi_i^{\ 2})$, as, at resonance $E_K = E_P$.

Now suppose material is removed from the horn where there is little motion but substantial stress, such as the regions about a motional node. The potential energy thus is decreased but, if the vibration amplitude is kept the same, the sum $\Sigma_i(m_i\xi^2_i)/2$ does not change. Then

$$\omega_1^{\ 2} = 2E_{P1}/\Sigma_i(m_i\xi_i^{\ 2}) = E_{p1}/[E_{K0}/\omega_o^{\ 2}]$$

where the subscripts 1 and 0 denote respectively the new resonant frequency resulting from the removal of material and the original resonant frequency.

We then have

$$(\omega_1/\omega_0)^2 = E_{P1}/E_{K0} = (E_{P0} - \Delta E_p)/E_{K0} = 1 - \Delta E_p/E_{K0}$$

Modal analysis provides both the resonant frequency, the amplitude and stress distributions in the unaltered horn permitting calculation of the potential energy in a small amount of mass undergoing little displacement in comparison to other parts of the horn and that is removed in the altered horn. The analysis is then repeated with this mass removed and a new resonant frequency computed. Since then both and ω_0 and ω_1 as well as ΔK_p are known, E_{K0} can be found as:

$$E_{K0} = \Delta E_P/[1 - (\omega_1/\omega_o)^2]$$

Having found the stored energy of vibration at a particular level of stress, the power dissipation ,P, in the horn follows, from the definition of Q, as

$P = f$(energy lost per cycle) = $2\pi f E_{K0}/Q$.

In a like manner, if some mass is added or subtracted from a horn where there is maximum motion and neglible stress, the stored energy of vibration can be found computing the kinetic energy, ΔK_e, of the mass removed from the modal analysis results for the unaltered horn as:

$$K_{E0} = \Delta K_e / [1 - (\omega_0 / \omega_1)^2].$$

Note that this analysis assumes that the Q is independent of stress. Using the value of Q appropriate to the largest value of stress in the horn will then provide an upper limit to power consumption.

Maximum safe operating stresses (strains) in ultrasonic horns.

Reference: http://www.modalmechanics.com/Acoustic_loss_at_substantial_ultrasonic_strain_in_6Al-6V-2Sn_and_sintered_6Al-4V_Titanium.htm

It has generally been determined, from stress versus cycles to failure data obtained from applying repetitive cyclic stress to specimens, that indefinite life in vibration can be obtained ***in metals*** by choosing the maximum operating stress to be no more than one third of the yield stress for the material. For example, the yield stress of 6Al-4V titanium is about 120,000 psi (827 MPa). Indefinite horn life can be achieved by ensuring that no part of the horn experiences more than 40,000 psi (275 MPa).

It is possible to also design plastic horns, as has been demonstrated by the technically, if not commercially, successful development of the ultrasonic tooth brush. For a material such as polycarbonate, the yield stress is about 9,000 psi (62 MPa). However, it has been found that above a 20 kHz cyclic stress of about 400 psi (2.7 MPa), which corresponds to a free end peak-peak displacement of a simple half wavelength rod of 0.6 mils (15 μ), failure occurs due to softening of the material caused by the elevation in temperature from cyclic stress loss. The Q of polycarbonate has been measured to be about 70! Nevertheless, its tested use as low frequency resonators, such as those operating in 50-150 Hz region where the power loss is reduced by low frequency operation, has shown that it may operate indefinitely providing the maximum cyclic stress is kept below 2,000 psi (1.4 MPa). The table below provides data on the low frequency cyclic endurance of some common thermoplastics.

Thermoplastic film	Thickness (inch)	Condition	Density (lbsf/in^3)	Modulus (psi)	Q
PC (polycarbonate)	0.022	annealed 250F 1 hour	0.04	260,000	70
PVC (polyvinylchloride)	0.015	cut from packaging	0.048	460,000	55
PET (polyethylene terathylate)	0.012	cut from packaging	0.047	860,000	100
PS (polystyrene)	0.013	cut from packaging	0.038	570,000	100

Thermoplastic film	Indefinite cyclic life fatique limit (psi)	Notes
PC (polycarbonate)	2,000	74 Hz, vane crack at 2300 psi
PVC (polyvinylchloride)	2,000	63 Hz, vane crack at 2500 psi
PET (polyethylene terathylate)	>4,500	93 Hz, cyanoacrylate adhesive failure, vane intact
PS (polystyrene)	3,500	99 Hz, vane crack at 3750 psi

Table 1

Effect of residual stress upon Q

Material for horns is commonly obtained in round rod or billet form. Stress induced during rolling or rerolling or by forging have been shown to substantially affect the Q and thus running power consumption. For 6Al-4V Titanium, chiming measurements of blanks and tests made upon horns finished from blanks showed that tips operating at 40,000 psi (275 MPa) peak stress made from blanks having a Q below about 7500 heated sufficiently to cause tensile failure.

Residual stress may be removed by annealing material prior to machining. In 6-4 Ti, annealing will, in general, restore the Q to its established level of about 20,000. Annealing of other Titanium alloys as well as the stainless steels is also recommended to obtain optimum performance.

Heat treatment to increase the yield strength of precipitation hardenable stainless steels does not produce the expected increase in free face displacement as many of these treatments, especially those for 15-7PH ,17-4 PH, Aeromet 100 and Custom 455, lower the Q and cause failure from an elevation in temperature during operation. The Q for all steels measured decreases, in any case, with strain. In designing stainless steel horns, it is best to choose materials with the highest yield strength in the annealed state and to disregard the higher strength available from heat treatment.