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Fundamentals of Ultrasonic Wave Propagation

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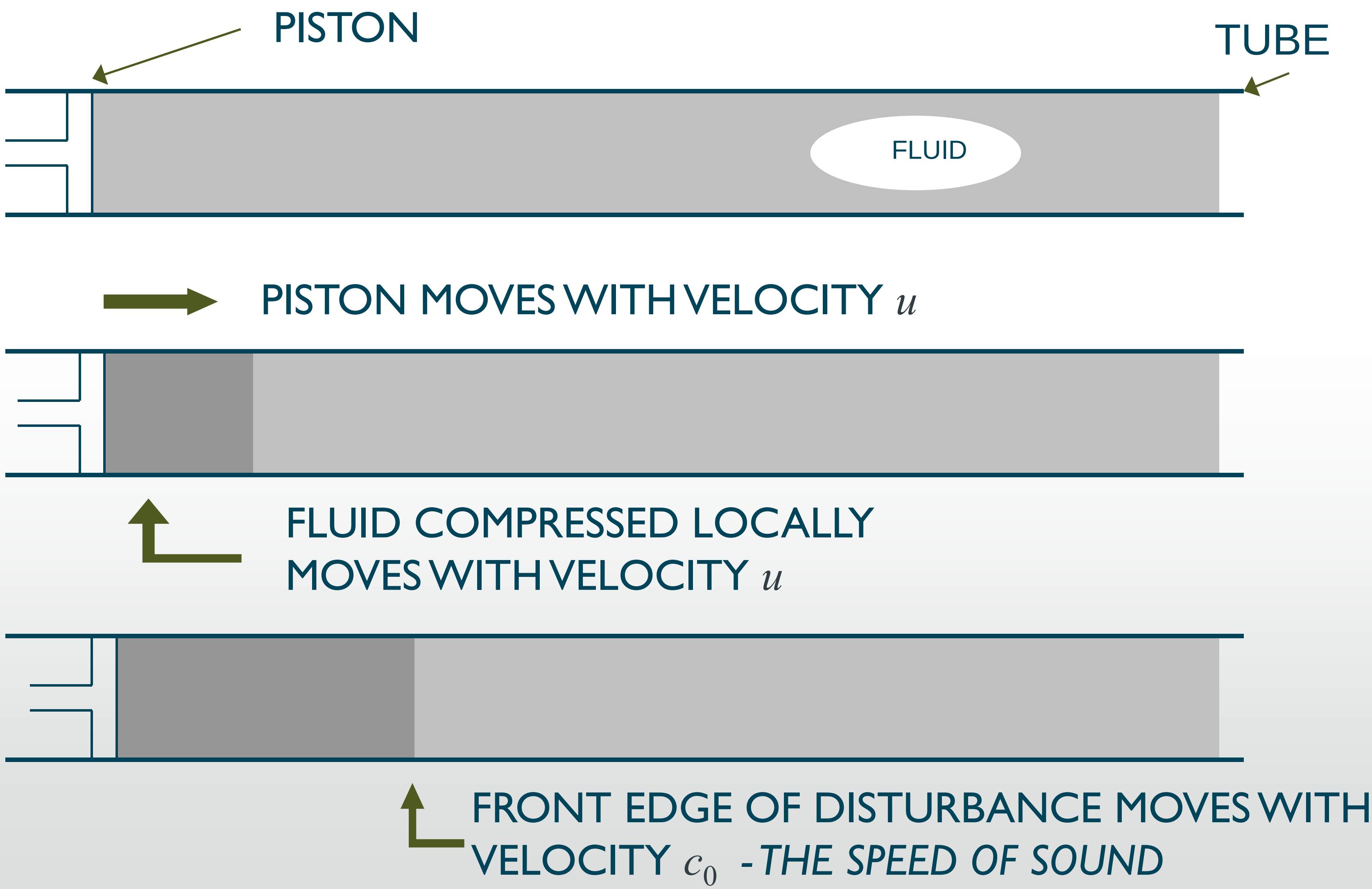
Aim

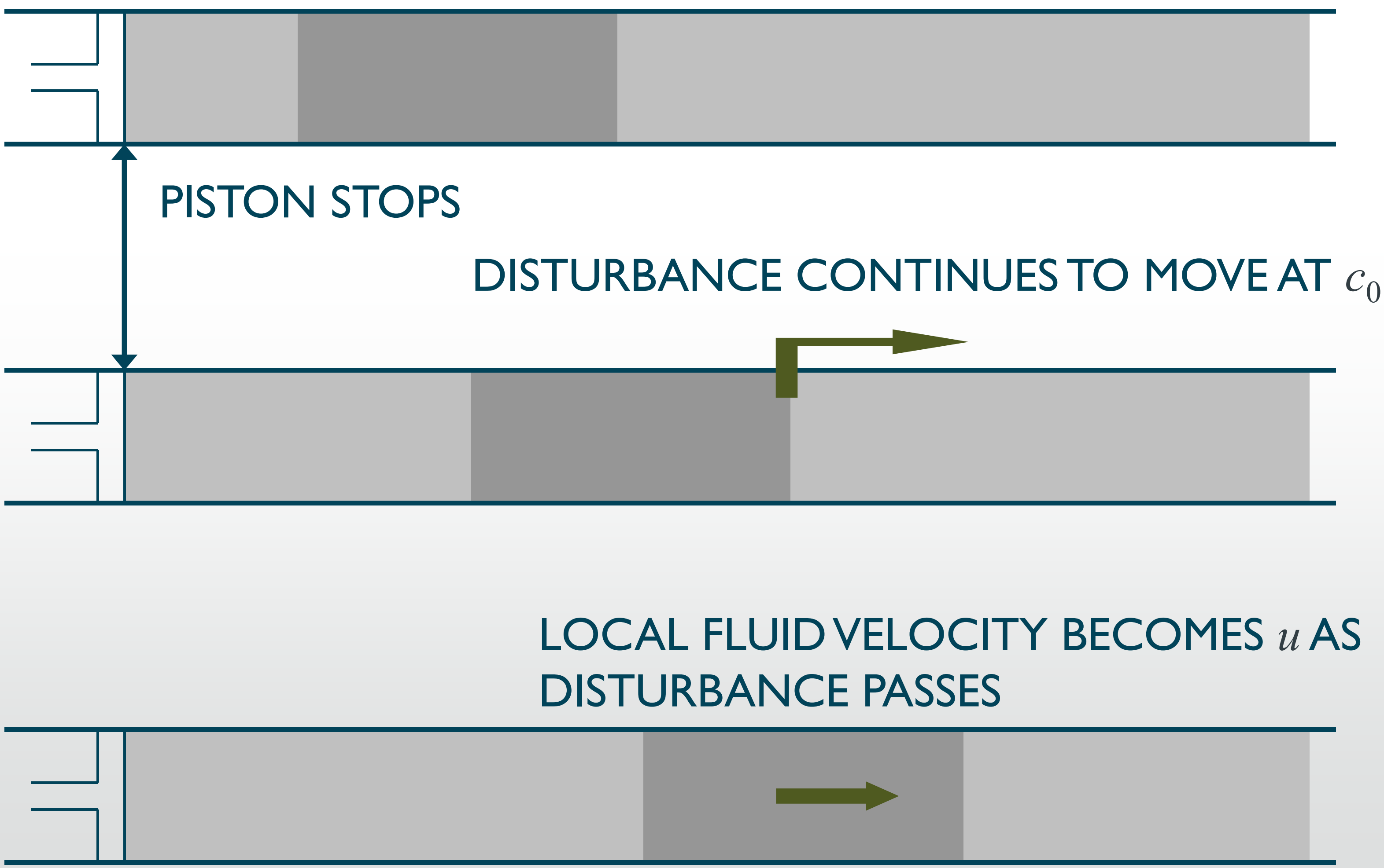
- To give a general overview of ultrasound propagation covering:
  - Basics
  - Linear propagation
  - Interfaces
  - Diffraction
  - Nonlinear propagation

Basics

Propagation of Acoustic Disturbances

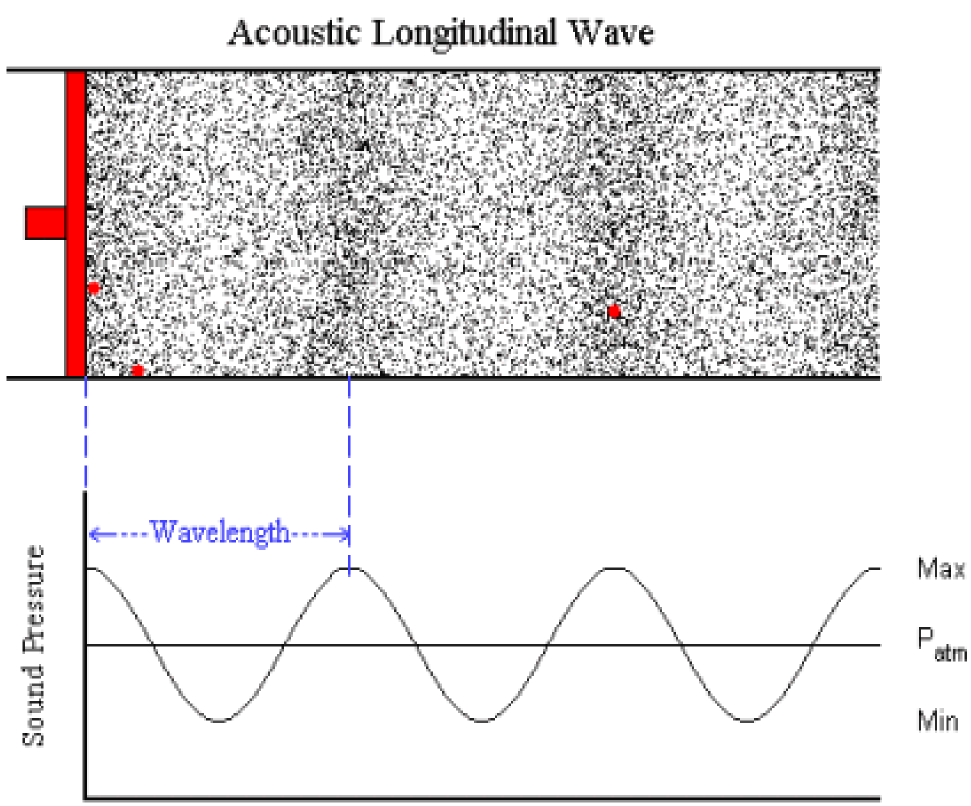


Propagation of Acoustic Disturbances



Basic Physical Characteristics

- In fluids only longitudinal ultrasound waves propagate
- Displacement of media is in direction of propagation
- Soft tissues have very low shear modulus



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Basic Parameters

- Particle velocity $\underline{u} = u_x \hat{x} + u_y \hat{y} + u_z \hat{z}$
- Total pressure P_T
- Total density ρ_T

When no sound is present:

$$\underline{u} = 0 \qquad \rho_T = \rho_0 \qquad P_T = P_0$$

With an acoustic wave:

$$\underline{u} \qquad \rho_T = \rho_0 + \rho \qquad P_T = P_0 + p$$

Acoustic component

Basic Equations

The acoustic wave equation is obtained by combining three equations:

1. The Continuity Equation (Conservation of Mass);
2. The Force Equation (Conservation of Momentum);
3. The Equation of State.

$$\frac{\partial \rho_{tot}}{\partial t} + \frac{\partial(\rho_{tot} u)}{\partial x} = 0$$

$$-\frac{\partial p_{tot}}{\partial x} = \left(u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} \right) \rho_{tot}$$

$$p_{\text{tot}} = f(\rho_{\text{tot}}) = P_0 + \frac{A}{\rho_0}(\rho_T - \rho_0) + \frac{B}{2\rho_0^2}(\rho_T - \rho_0)^2 + \ldots$$

The One Dimensional Wave Equation

Linearize these equations by assuming that $u << c_0$ and $\rho << \rho_0$. Also use fact that ρ_0 doesn't vary in space or time

Can then derive the Wave Equation in 1-D:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = 0$$

For a plane wave:

$$p = \rho_0 c_0 u$$

→ *The product of equilibrium density ρ_0 and speed of sound c_0 is known at the characteristic acoustic impedance Z.*

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Values

- Consider two cases of a single frequency wave:
  - An airborne ultrasonic source producing a signal at 137 dB re 20 μPa;
  - A 3.5 MHz ultrasonic imaging system producing 5 MPa at focus.

Pressure Amplitude /MPa	Frequency /MHz	Medium	Displacement Amplitude / m	Velocity Amplitude /ms^{-1}	Acceleration Amplitude /ms^{-2}
0.0002	0.04	Air	1.9×10^{-6}	4.9×10^{-1}	$1.2\times10^{+5}$
5	3.5	Water	1.5×10^{-7}	3.3	$7.3\times10^{+7}$

Acoustic Intensity

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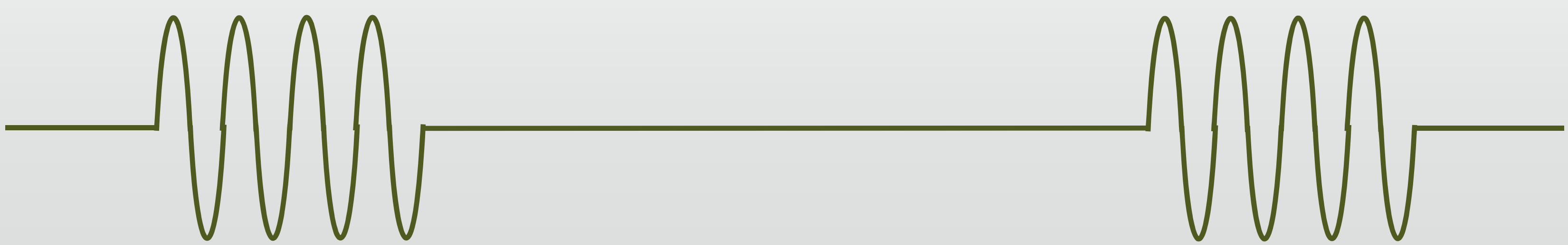
- The average rate of flow of energy through a unit area normal to direction of propagation.

$$I = \frac{1}{2}\frac{P^2}{\rho_0 c_0}$$

- ➔ Strictly for plane waves. P is acoustic pressure amplitude.
- ➔ Differences will occur in fields where the particle velocity and pressure are not in phase, but this is not normally significant.
- ➔ For pulsed waveforms need to allow for the mark-to-space ratio in calculating the time averaged intensity I_{TA}



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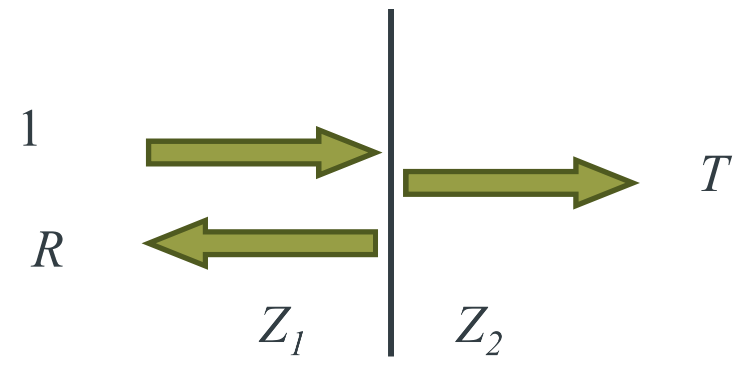
Interfaces

Interfaces

- At a boundary between two fluid media some of the energy will be transmitted and some reflected
- The extent of transmission depends on
  - Relative characteristic acoustic impedance of media
  - Angle of incidence



- e.g. for normal incidence the Amplitude Transmission and Reflection coefficients are given by

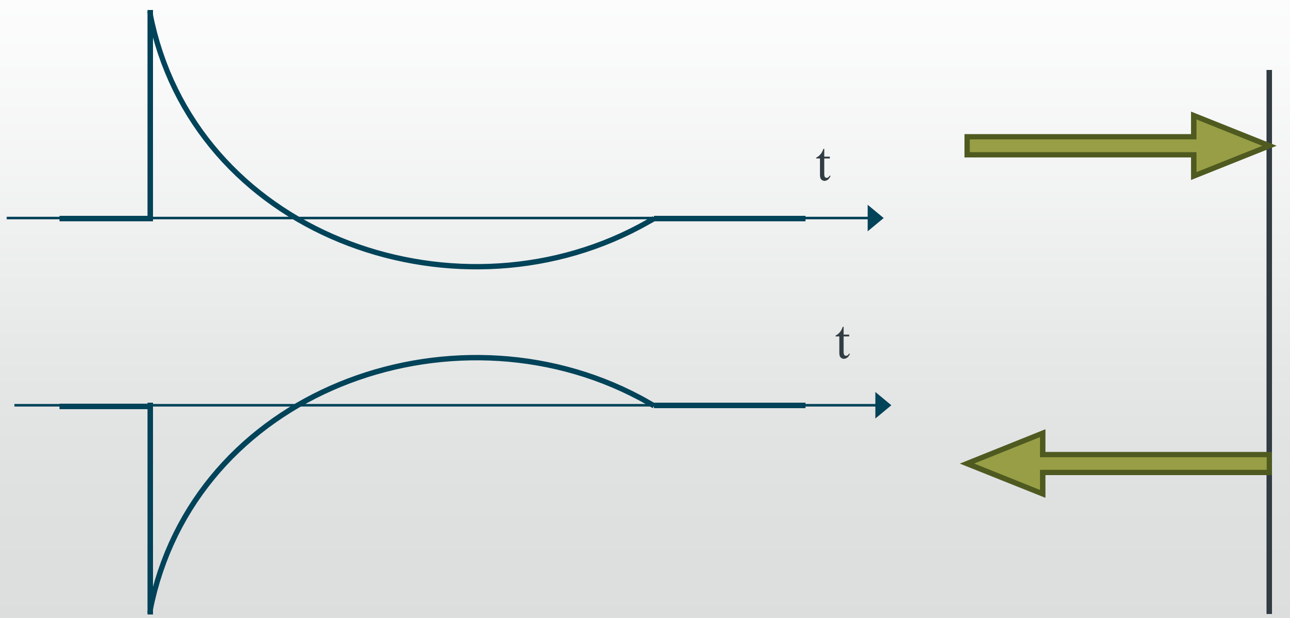
$$T = \frac{2}{(1 + Z_1/Z_2)}$$

$$R = \frac{(Z_2 - Z_1)}{(Z_2 + Z_1)}$$

→ *Note that in going from a low impedance material to high impedance the pressure may increase (but the displacement decrease)!*

Interfaces

- At a boundary between high and low impedance materials ($Z_1 >> Z_2$) the reflected pressure wave is inverted.
- For asymmetric waveforms (such as those generated by nonlinearity) this can turn a high amplitude compression in to a high amplitude tension.



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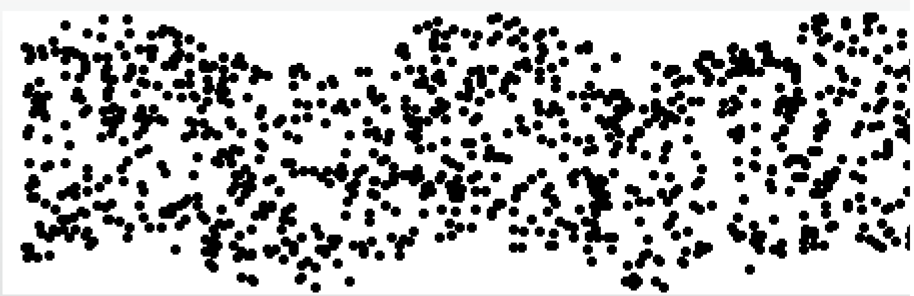
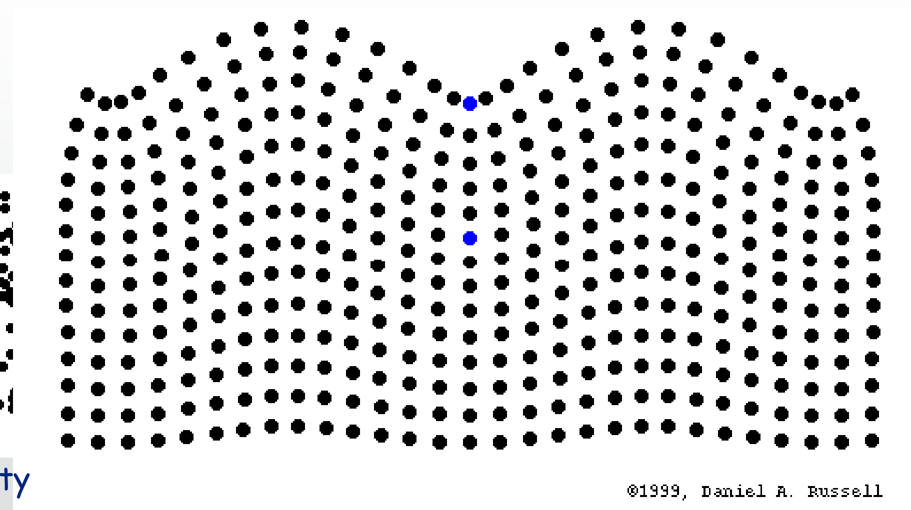
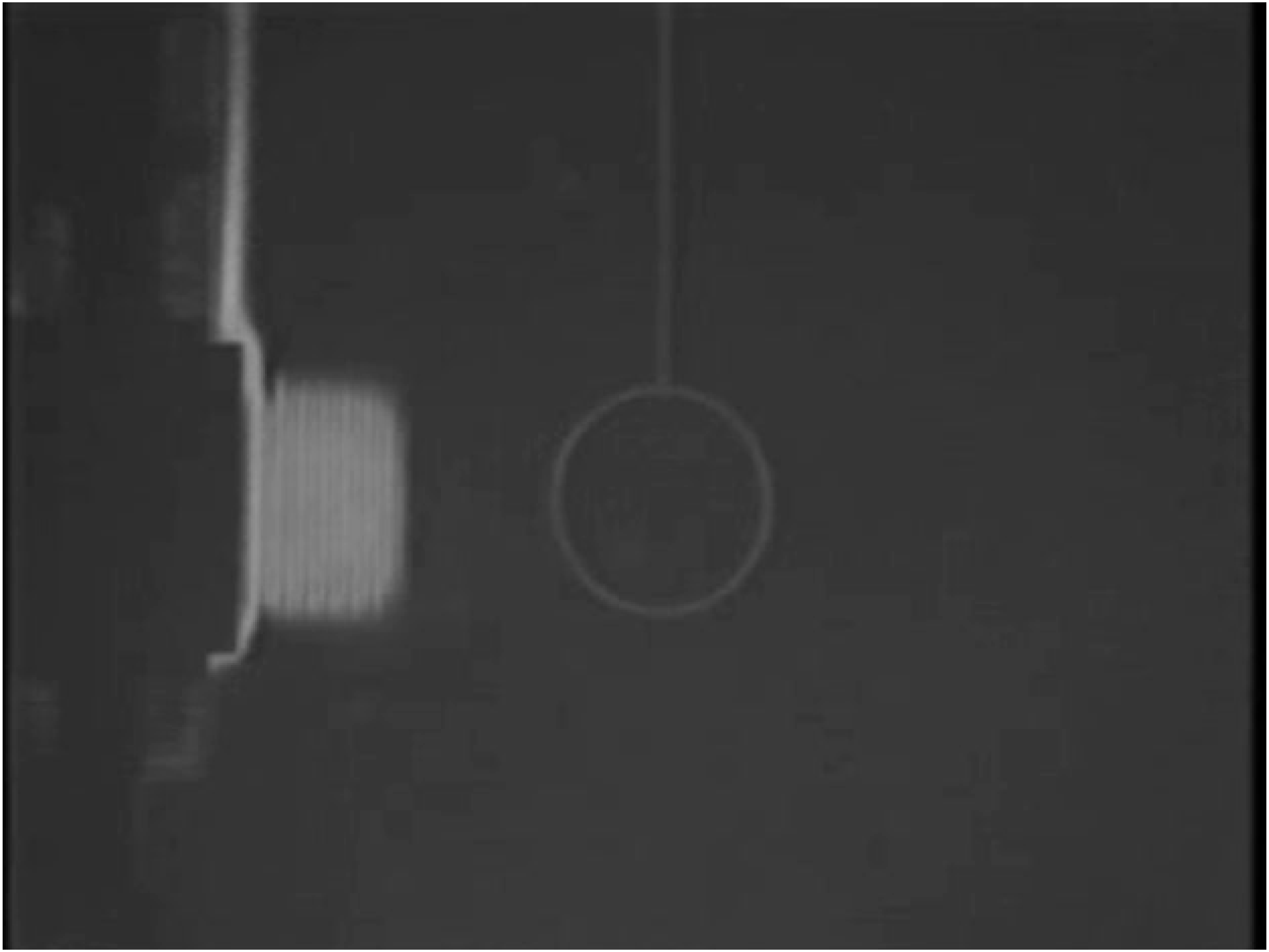
Mode Conversion

- At a boundary between a fluid and solid it is possible to excite shear waves and surface waves





Animations courtesy of Dr. Dan Russell, Kettering University



Transducer Fields

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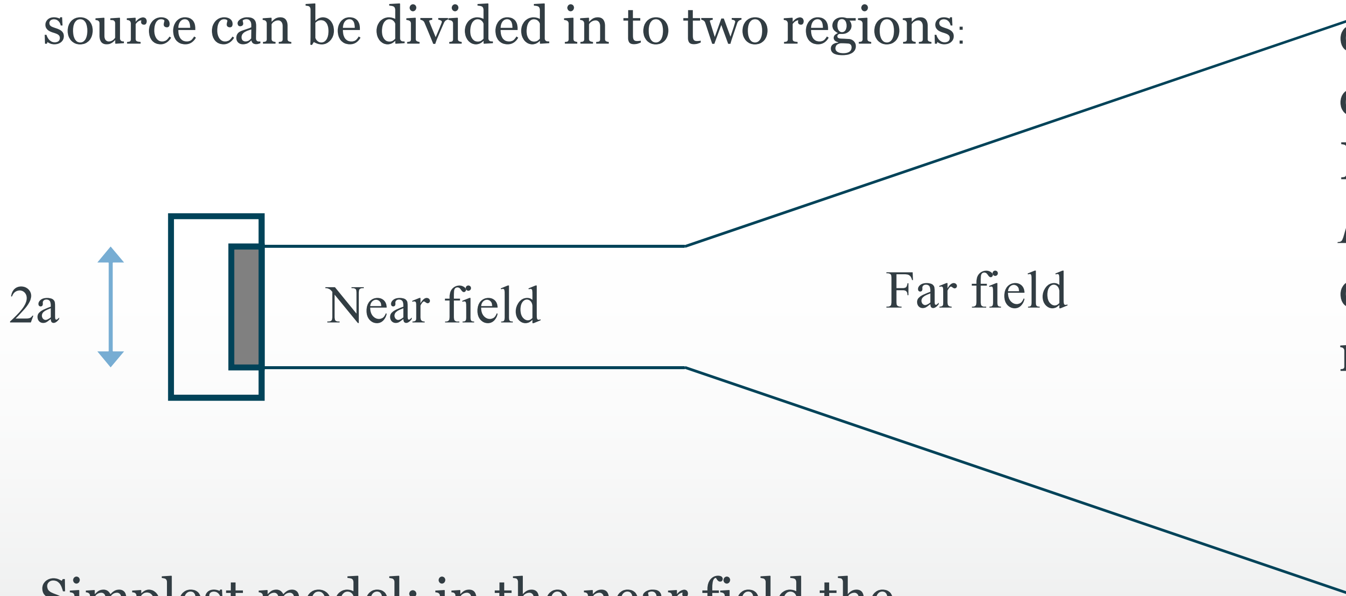
Near Field and Far Field of a Planar Source

Can consider that the field of a planar ultrasonic source can be divided in to two regions:

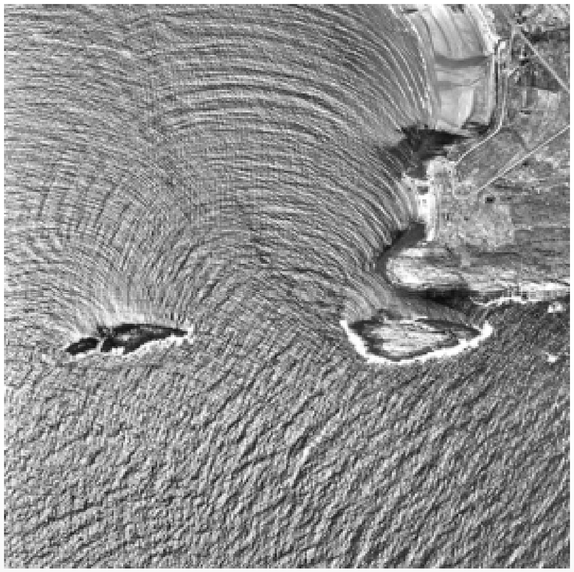


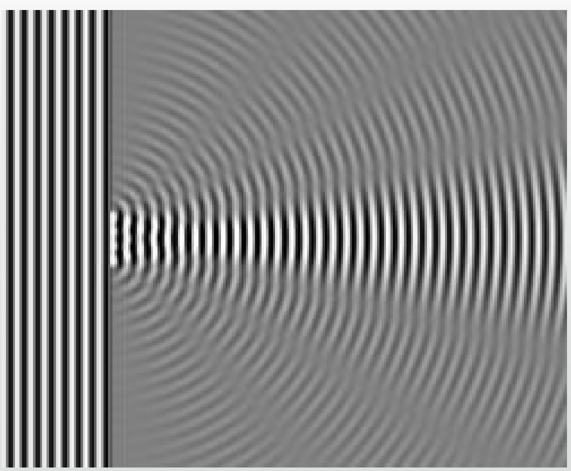
Transition can be considered to occur at the Rayleigh Distance R_0 where for a circular source of radius *a*:

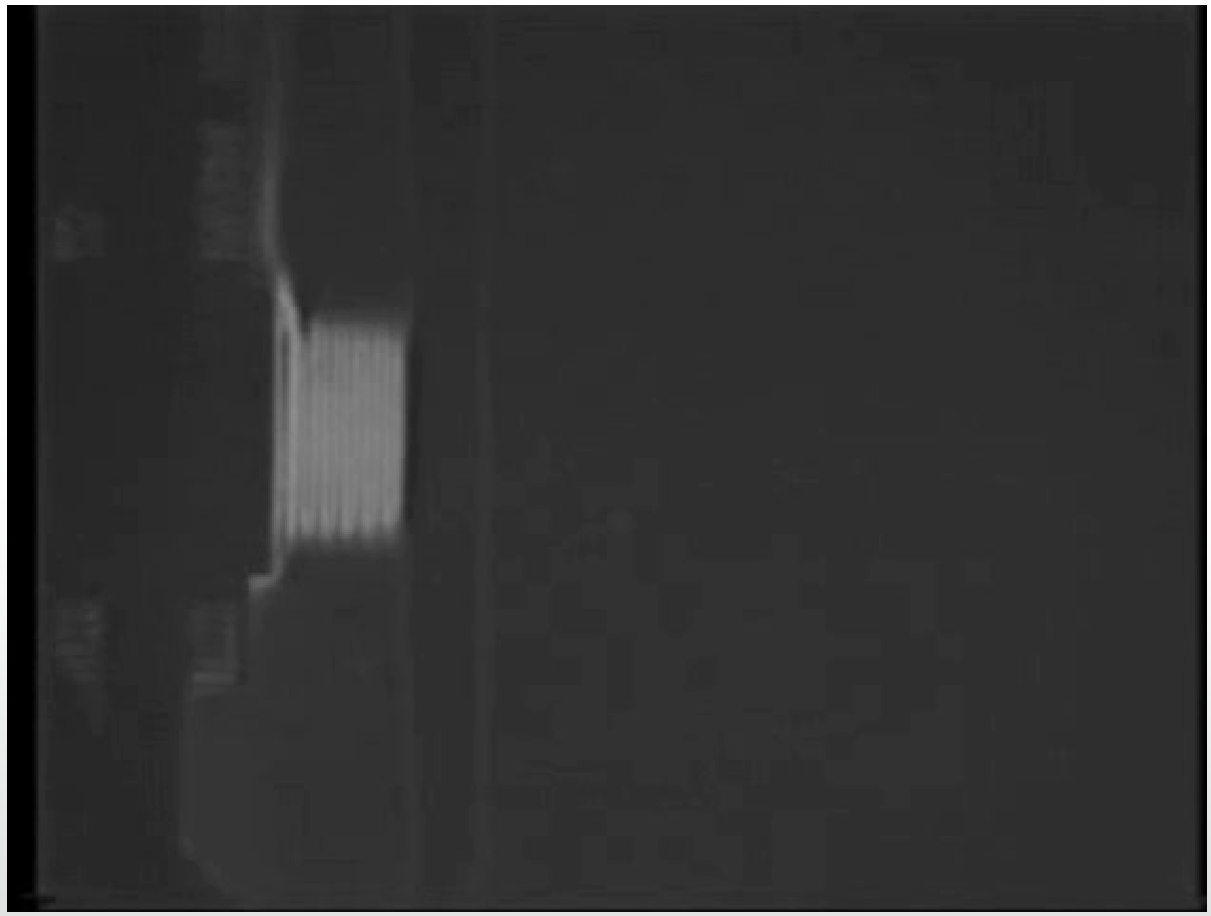
$$R_0 = \frac{\pi a^2}{\lambda} = \frac{k a^2}{2}$$

Simplest model: in the near field the propagation is planar (and collimated) while in the far field it is spherically spreading.

Diffraction







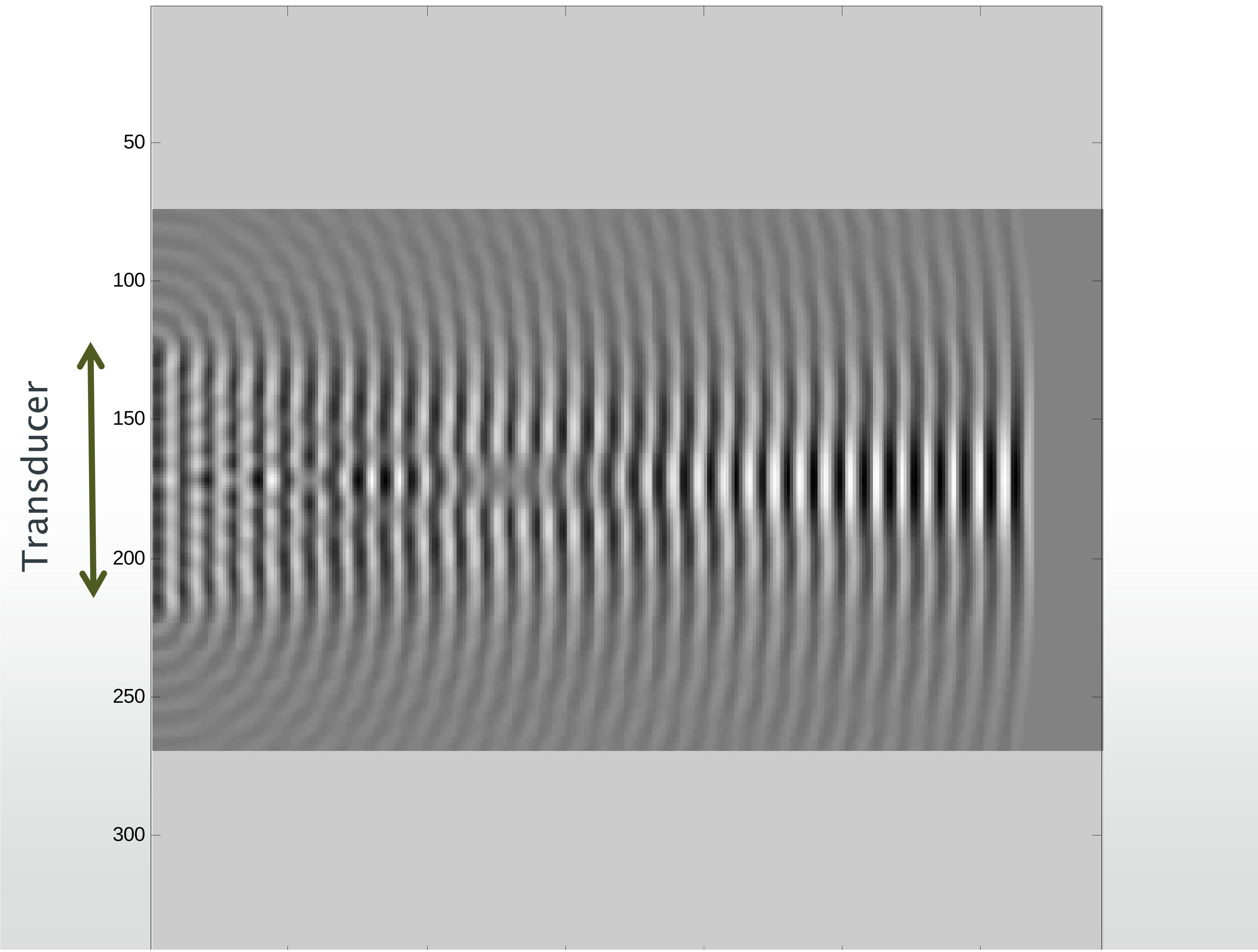
The Radiated Field: The Rayleigh Integral

- The total complex pressure generated by a vibrating source can be evaluated by integrating over the contributions due to all the surface elements.
- The ***RAYLEIGH INTEGRAL***

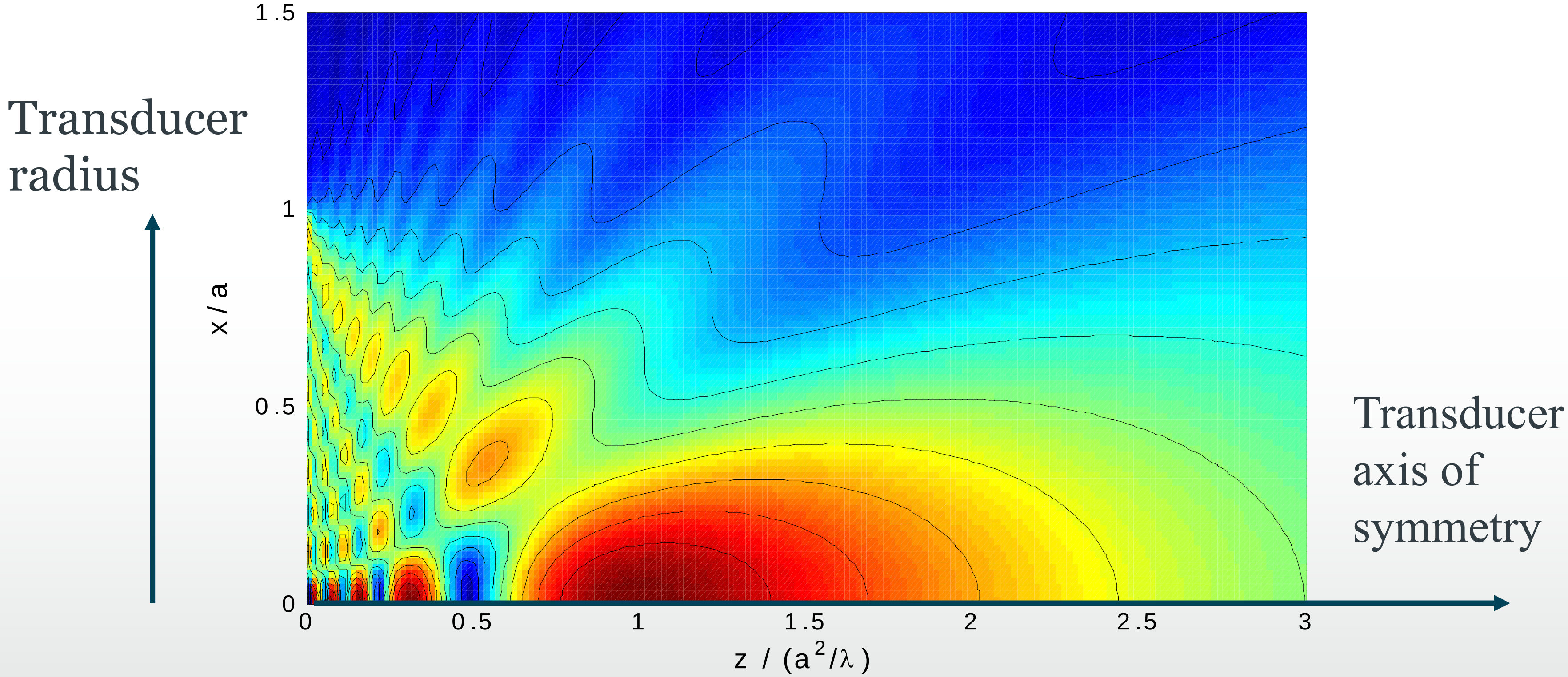
$$p(\mathbf{x}) = \int_S dp(\mathbf{x}) = \int_S \frac{j\omega\rho_0 U e^{-jkr}}{2\pi r} dS$$

- which in principle enables the calculation of the sound field produced by any distribution of complex normal velocity U of an otherwise rigid infinite plane boundary.


Transducer


50
100
150
200
250
300
Transducer

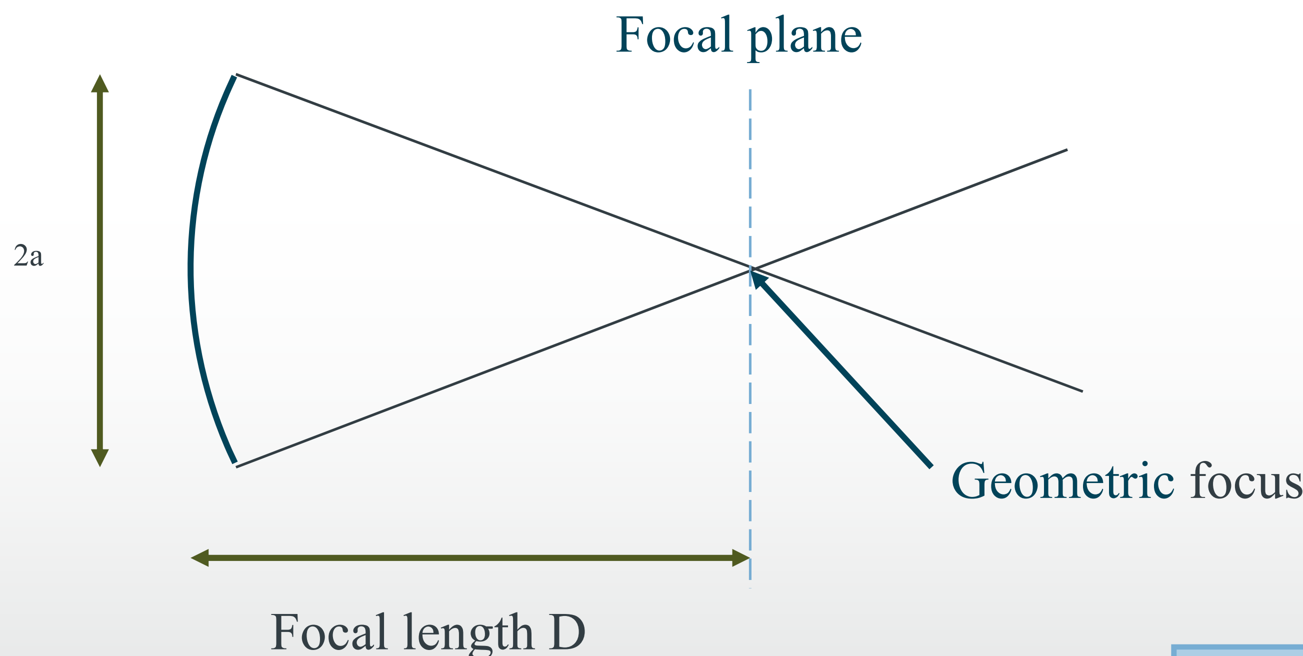
Near Field of a Circular Piston Radiator



The minimum beam diameter is only ~1/4 transducer diameter (measured at -3 dB level).

Focusing

In order to produce a narrower beam focus the ultrasound field:



The amplitude gain *G* is given by:

$$G = \frac{R_0}{D} = \frac{\pi a^2}{\lambda D}$$

Focused Circular Piston

- Weak: $0 < G < 2$
- Medium: $2 < G < 2\pi$
- Strong: $2\pi < G$

Axial variation:

$$p(z) \cong p_0 G \frac{D}{z} \frac{\sin X}{X}$$

$$X = \frac{G}{2}\left(\frac{D}{z} - 1\right)$$

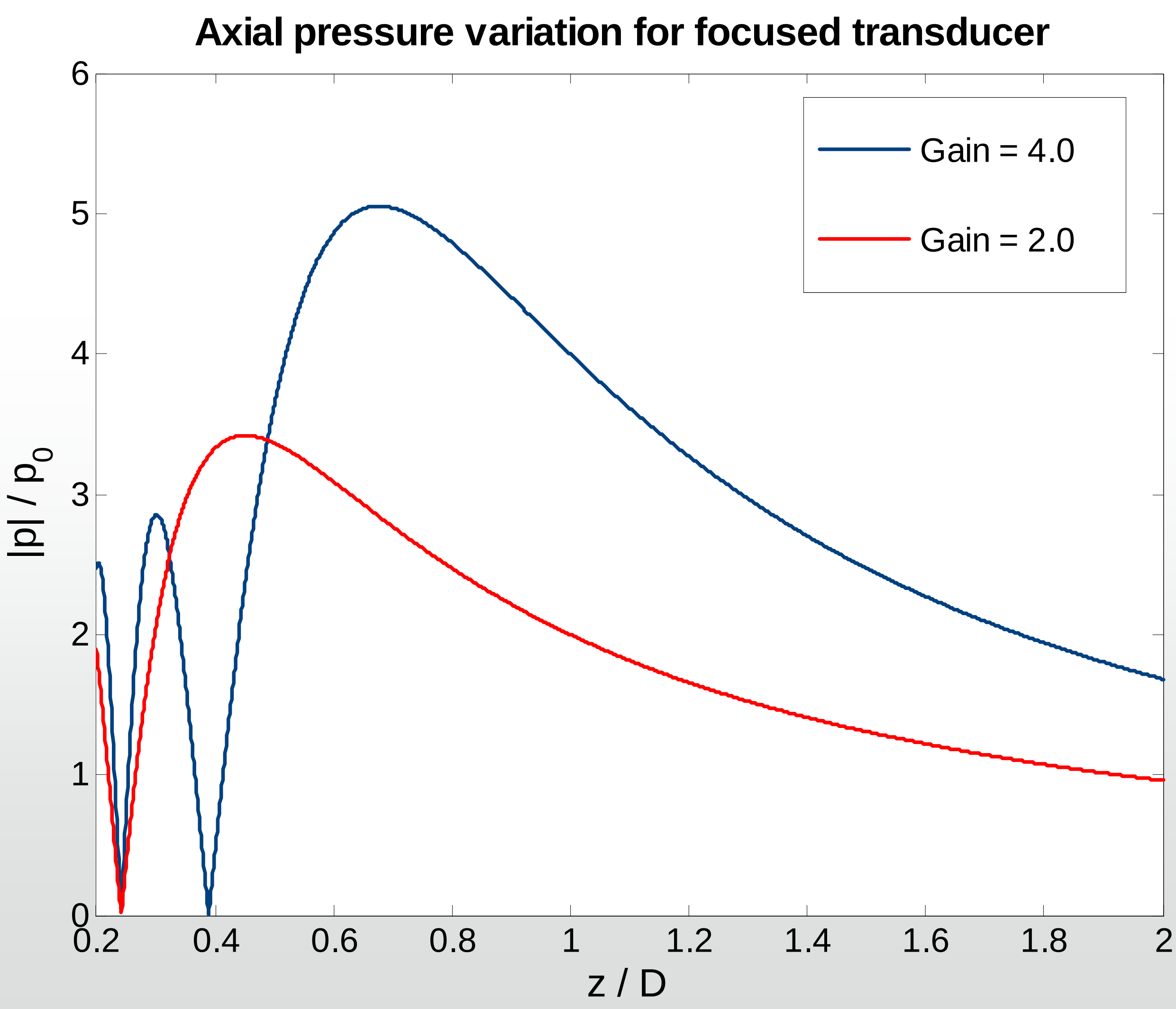
- At geometric focus:

$$p(D) = p_0 G$$

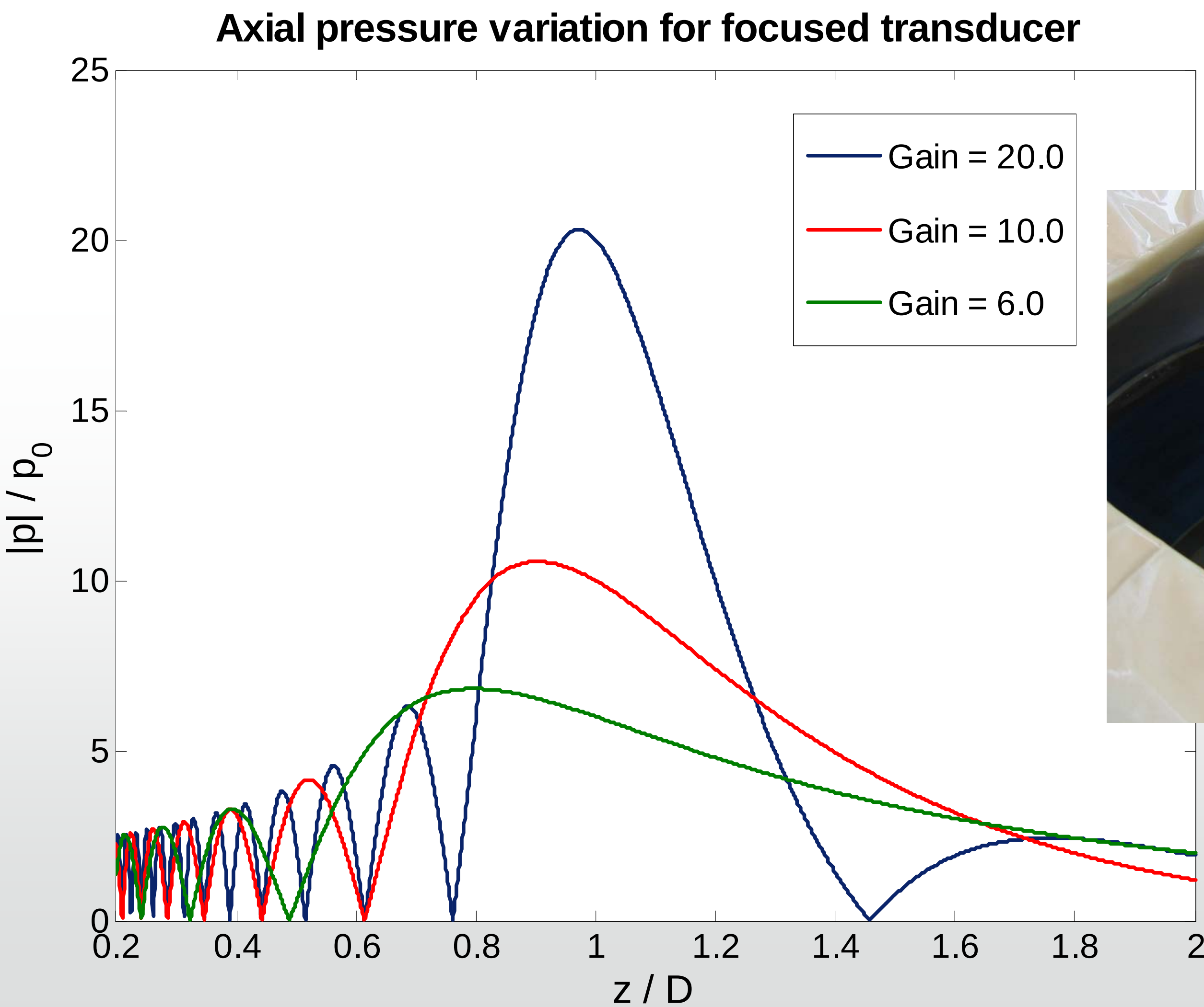
- -3 dB beam full width in focal plane:

$$x_{-3dB} = \frac{1.62a}{G}$$

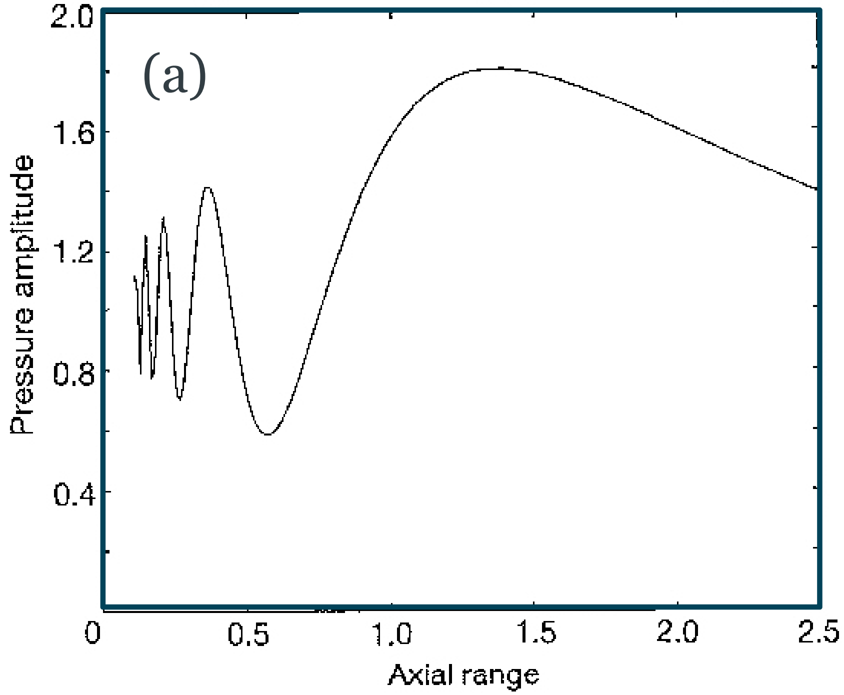
Focused Field Axial Variation

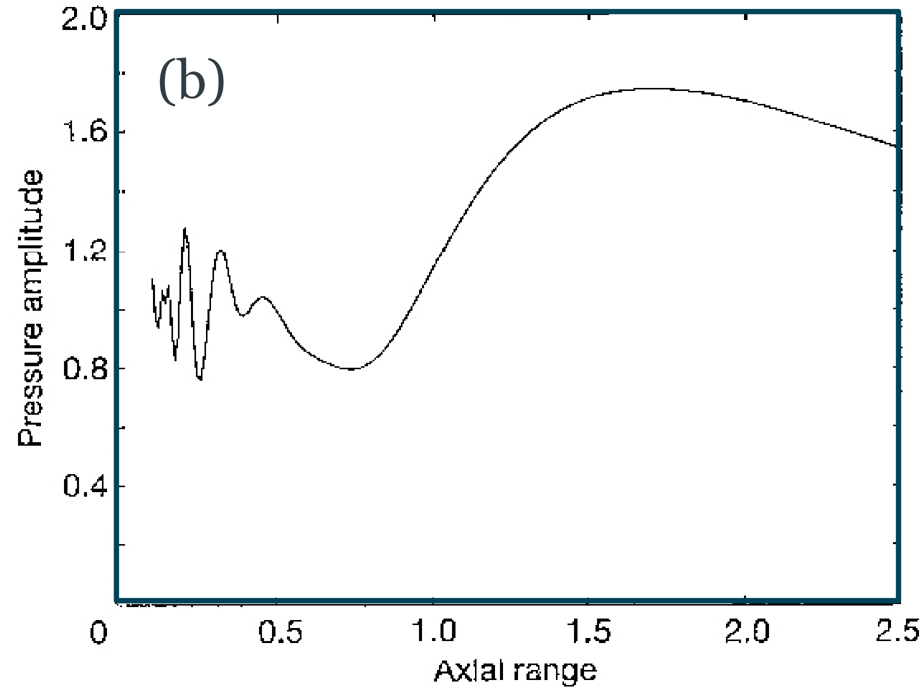


Focused Field Axial Variation



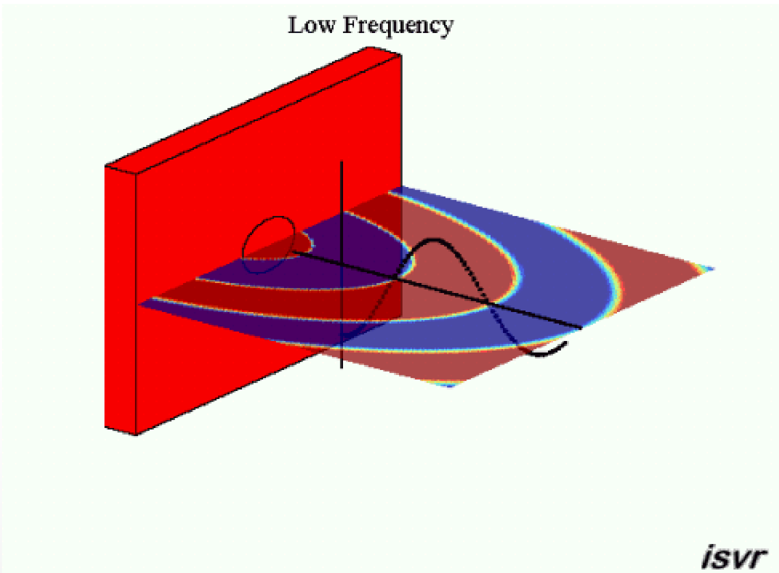
Rectangular Transducers

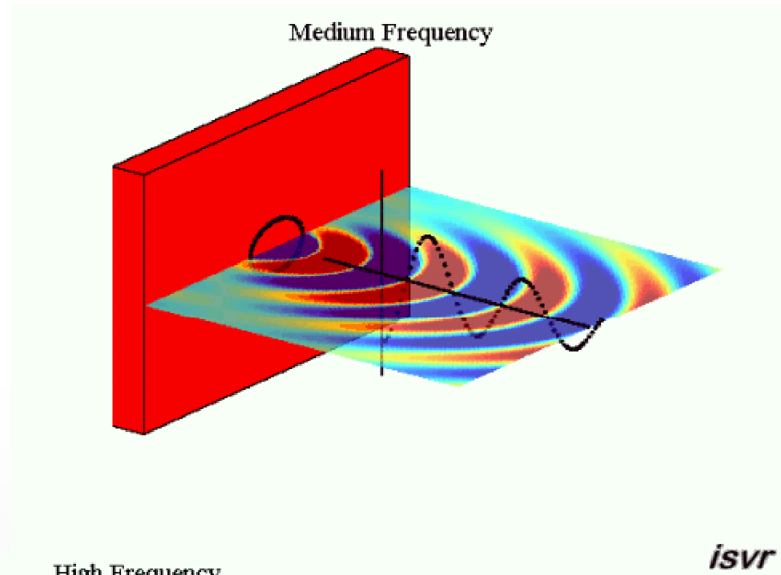


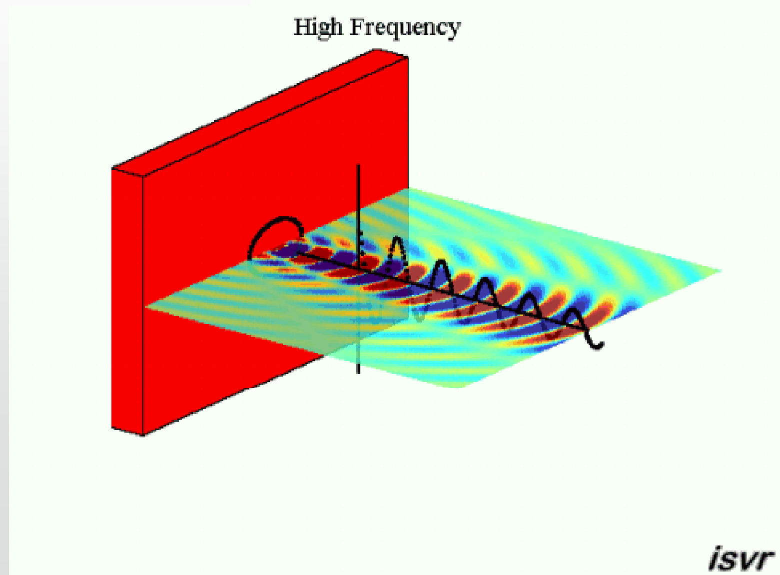


Axial variation of normalised acoustic pressure amplitude (p/p_0) for: (a) a square transducer; (b) a rectangular transducer with 1:2 aspect ratio.

The Far Field Pressure Distribution







$$p(\mathbf{x}) \sim \left[\frac{2\,\mathrm{J}_1(ka\sin\theta)}{ka\sin\theta}\right]$$

First zero occurs when $ka\,\sin\theta = 3.83$.

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Non-linear Propagation

Non-linear Propagation

- If the acoustic amplitude is sufficiently high then non-linear effects will become significant.

- A point on the wave with particle velocity u will travel with velocity

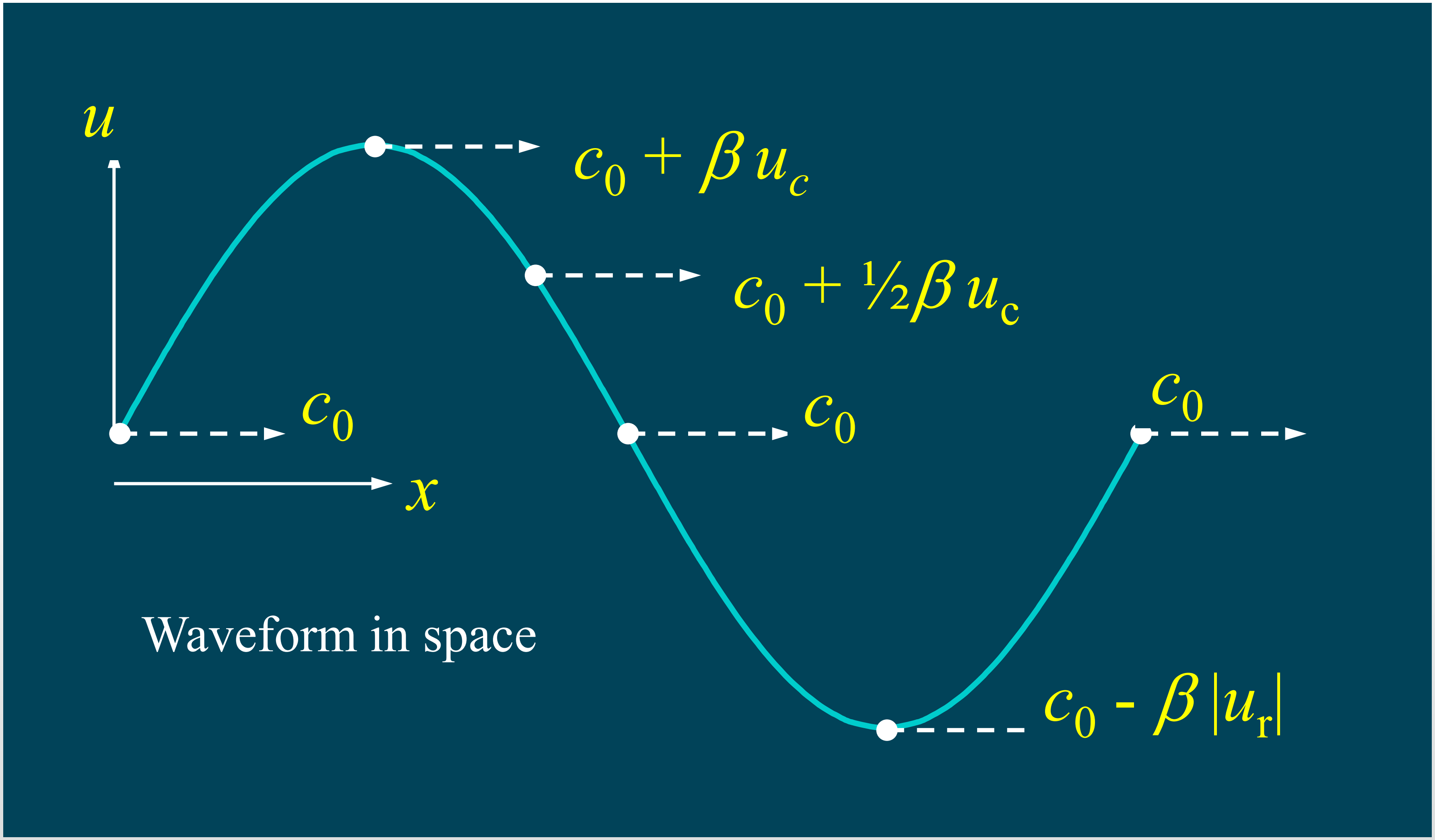
$$c_0 + \beta u$$

where the coefficient of non-linearity β:

$$\beta = 1 + \frac{B}{2A}$$

- Values for B/A vary from 5.0 for water to 6.3 for blood, ~ 6 - 7 for liver and ~ 10 for fatty tissue.

Propagation of Plane Wave



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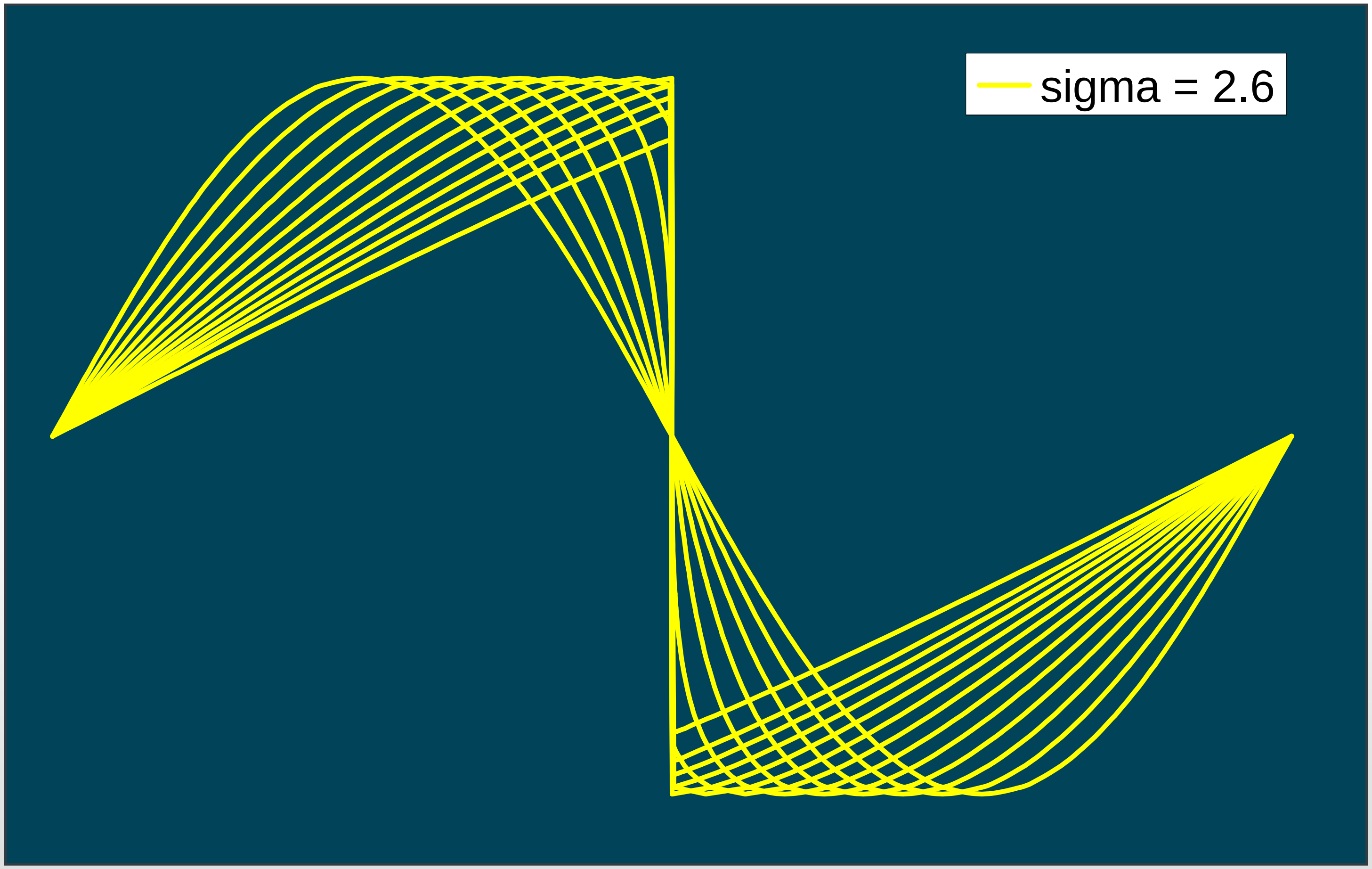
Non-linear Plane Wave Propagation



Non-linear Propagation

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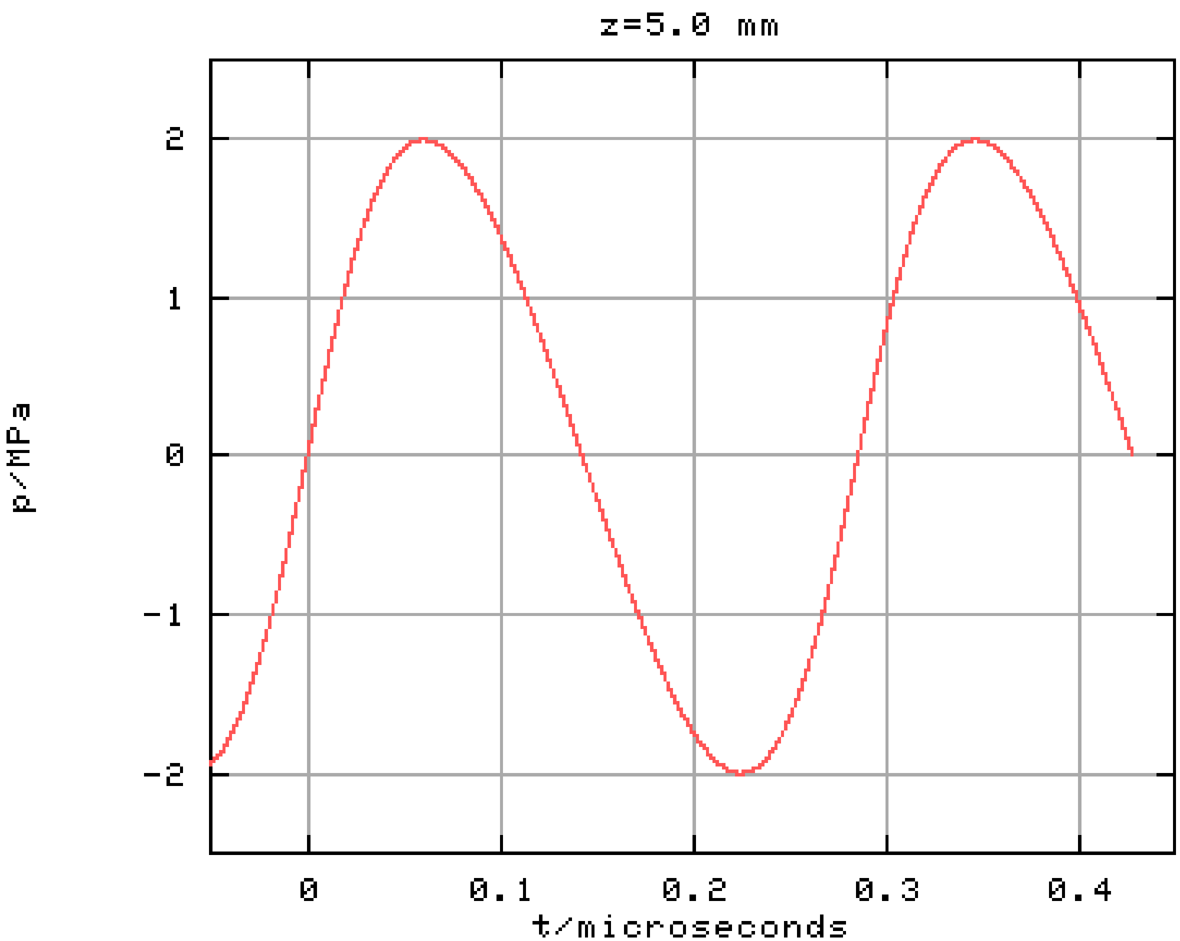

Results in:

- Generation of harmonics
- Generation of shock fronts
- Enhanced attenuation
  - Enhanced heating
  - Enhanced streaming
- Saturation

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The Shock Parameter σ

A measure of the extent of non-linear propagation

For a plane wave: $\sigma = \beta \varepsilon k z$

$\varepsilon = u_0 / c$	Acoustic Mach number
u_0	Peak particle velocity at source
$k = 2\pi / \lambda$	Wavenumber
z	Distance travelled
$\sigma = 1$	corresponds to shock front just forming
$\sigma = \pi/2$	corresponds to a full shock

Shock Distance for a Plane Wave

- The shock parameter $\sigma = 1$ corresponds to a shock front just forming.
- At high frequencies the plane wave shock distance can be small.
- So for example in water:

$$\beta = 3.5$$
$$f_0 = 3.5\ \text{MHz}$$
$$p_0 = 1\ \text{MPa}$$

$$\text{Shock distance} = 43\ \text{mm}$$

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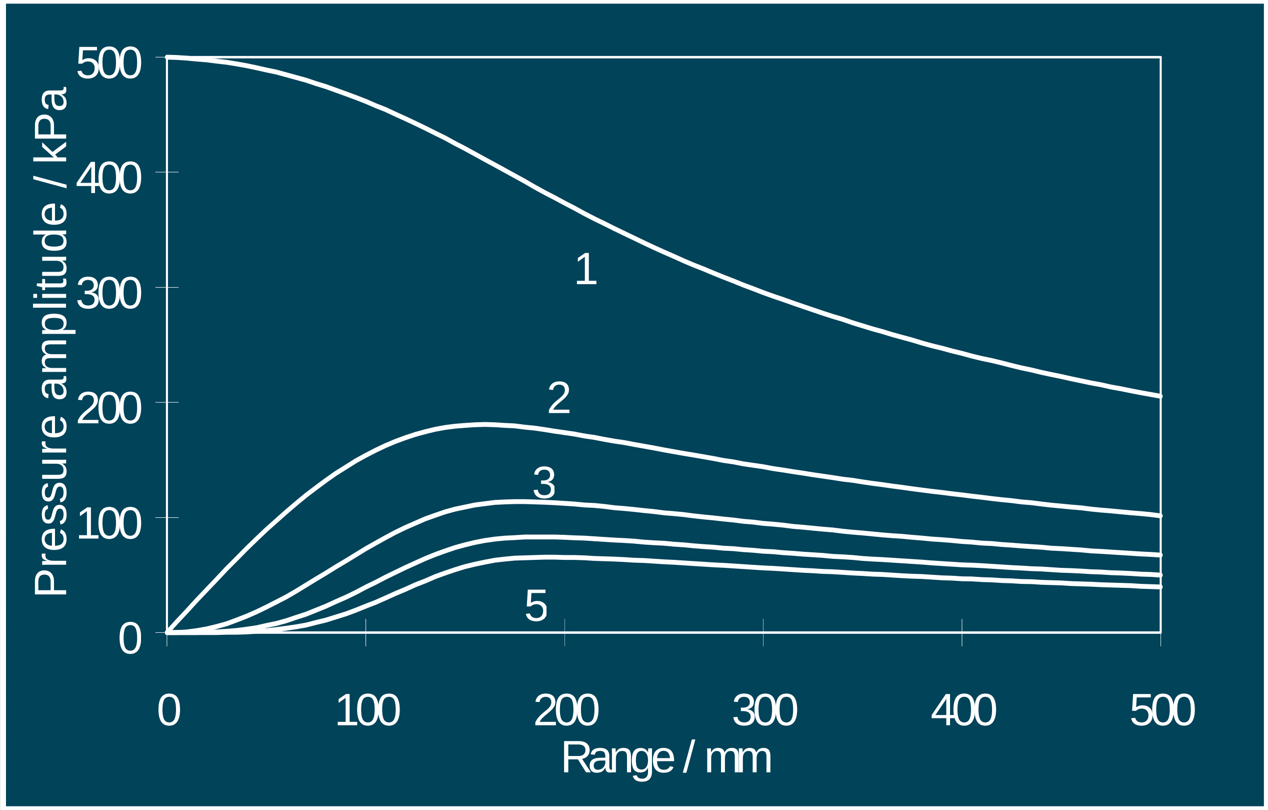
Plane Wave: Non-linear Propagation



Fundamental and second to fifth harmonics for a nonlinear plane wave in water. (f_0 = 3.5 MHz, P_0 = 500 kPa, G = 38).

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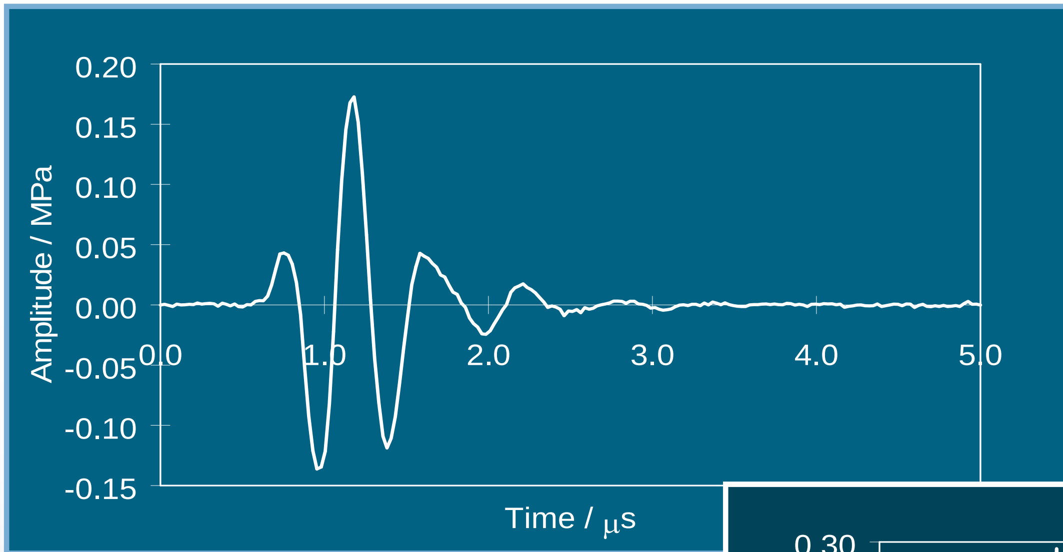
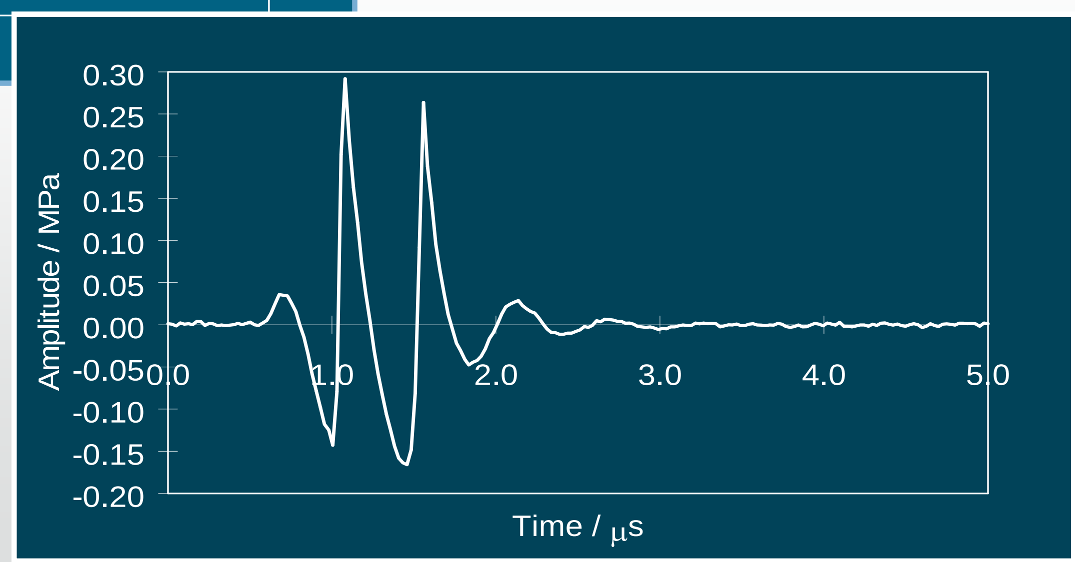
Initial and Distorted Pulses





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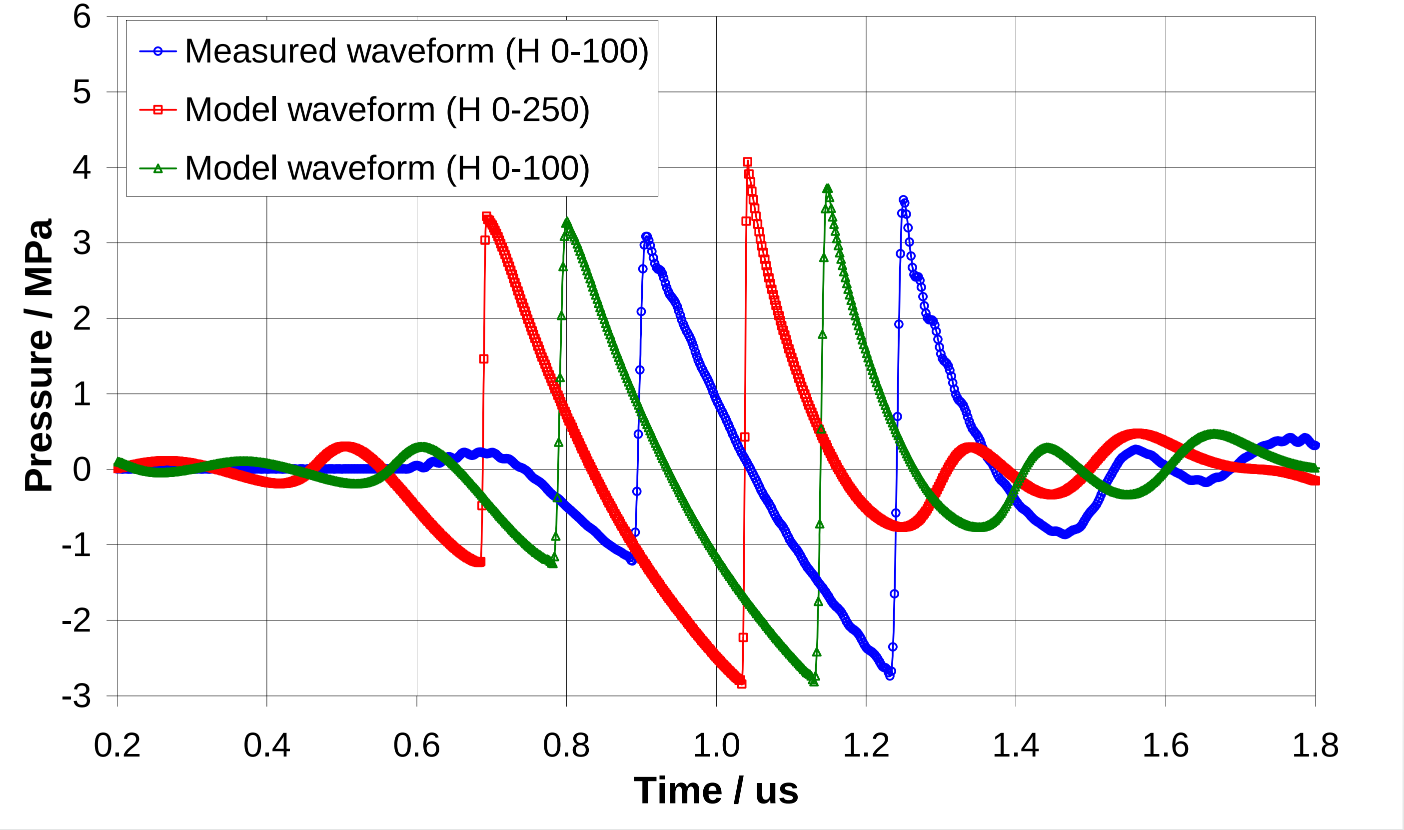
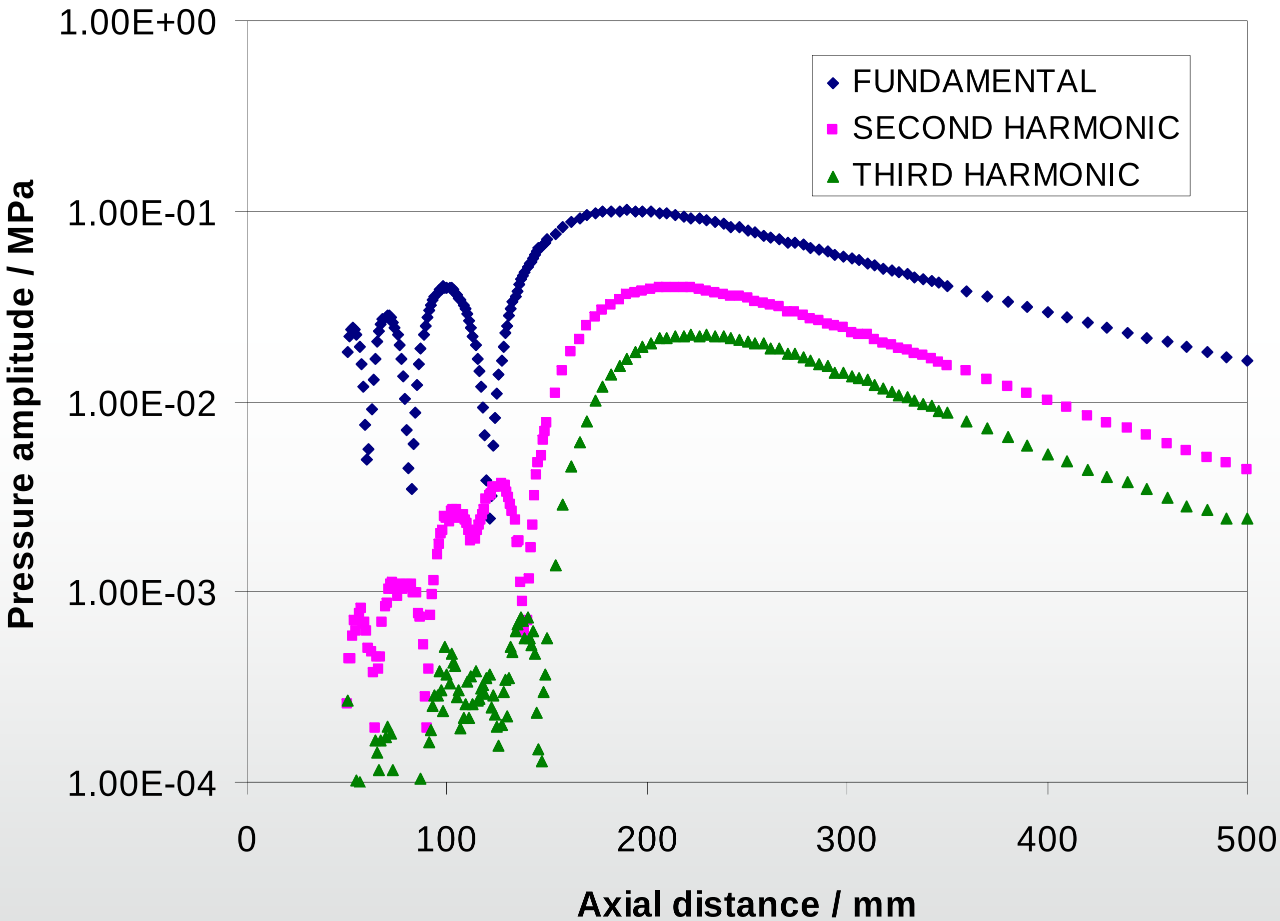
Measured Output and Model Prediction



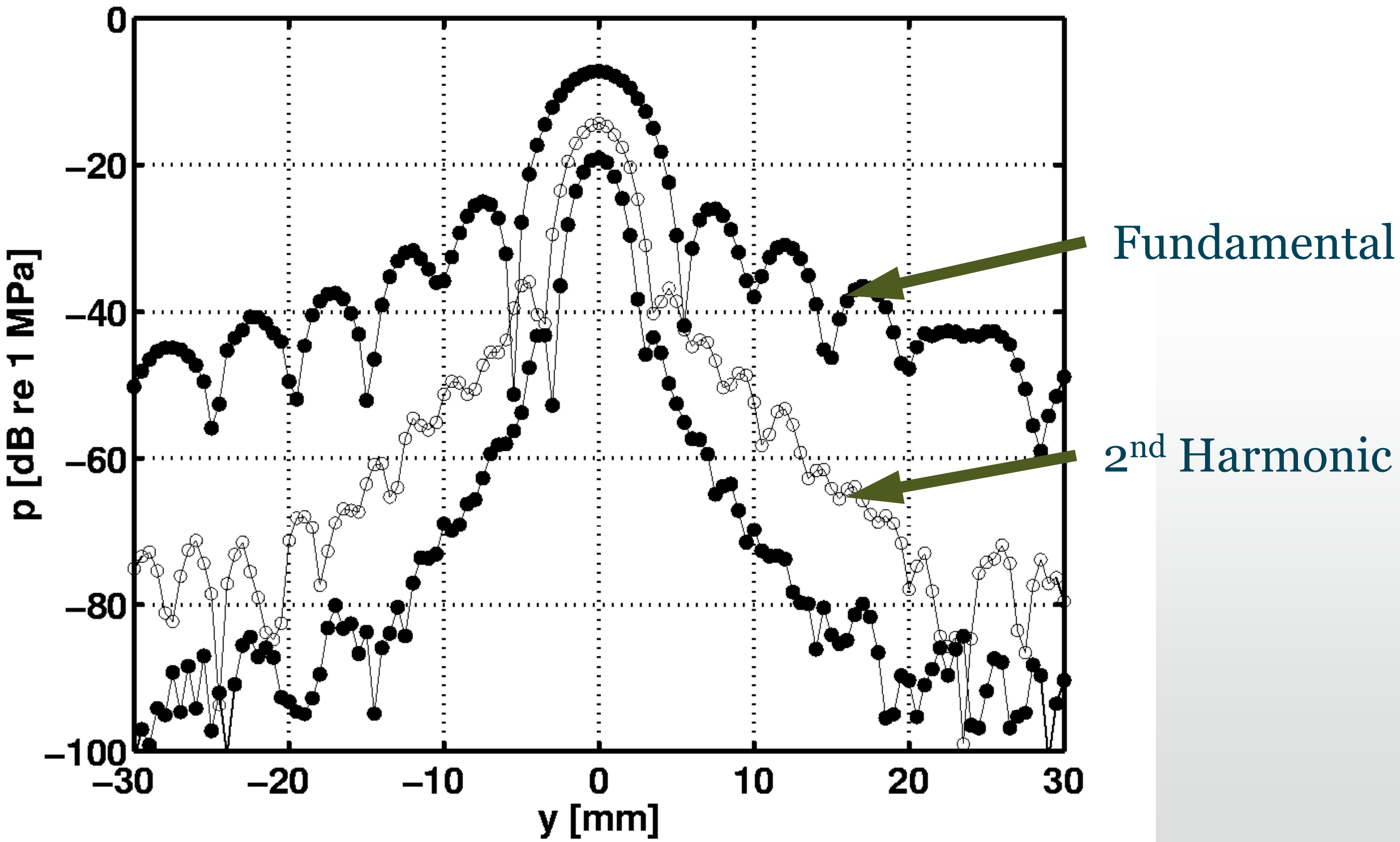
On axis time waveforms at z = 54 mm for a 3.1 MHz array in water. Model with 250 MHz filter (red squares); model with 100 MHz filter (green triangles) and experiment with 100 MHz filter (blue circles).

Focused Field (G = 8)



Axial variation for p_0 = 72 kPa, f_0 = 2.25 MHz and G = 8.0.

Beam Profiles for Demonstration Harmonic Imaging System



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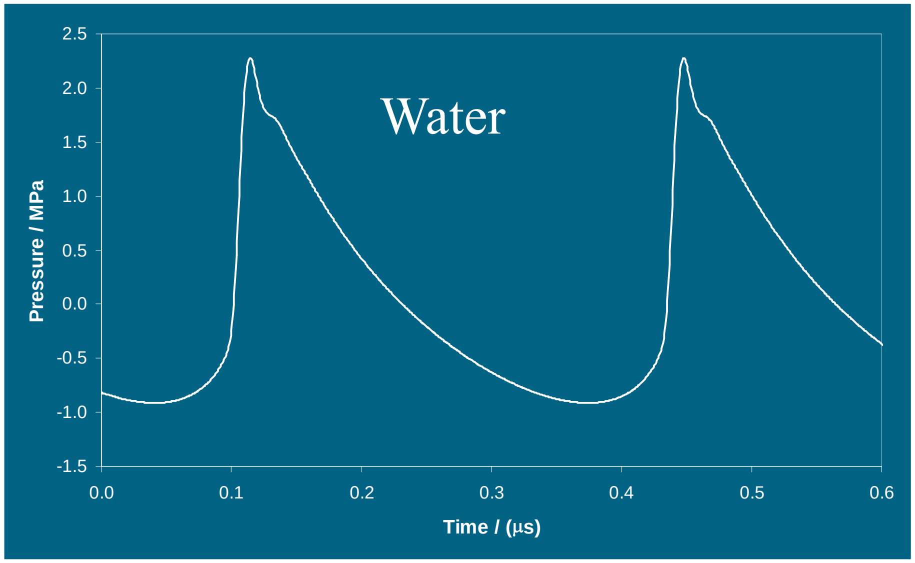
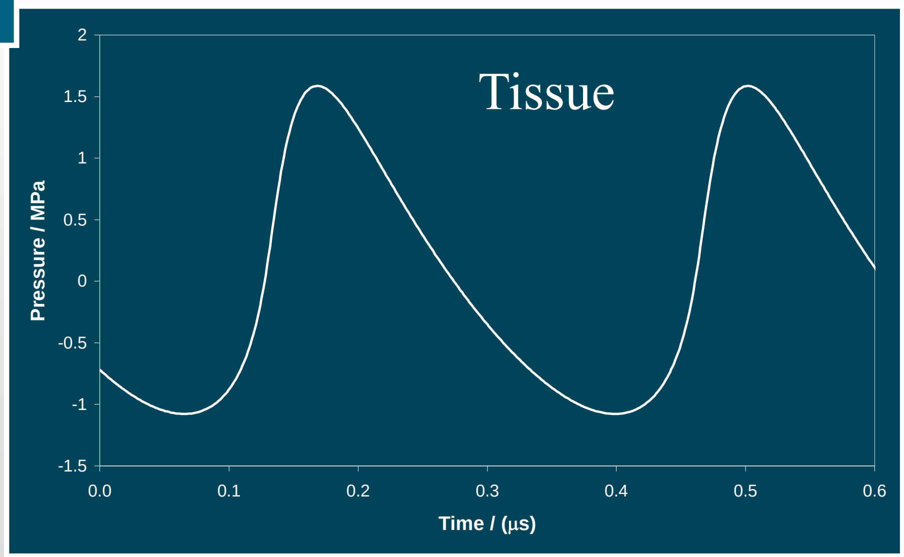
Predicted Non-linear Distortion



Predicted waveform at a range of 54 mm in water and tissue (attenuation 0.3 dB/cm/MHz) produced by an array 15 mm by 10 mm in size, with focal lengths of 80 mm and 50 mm. The source pressure amplitude is 0.5 MPa.



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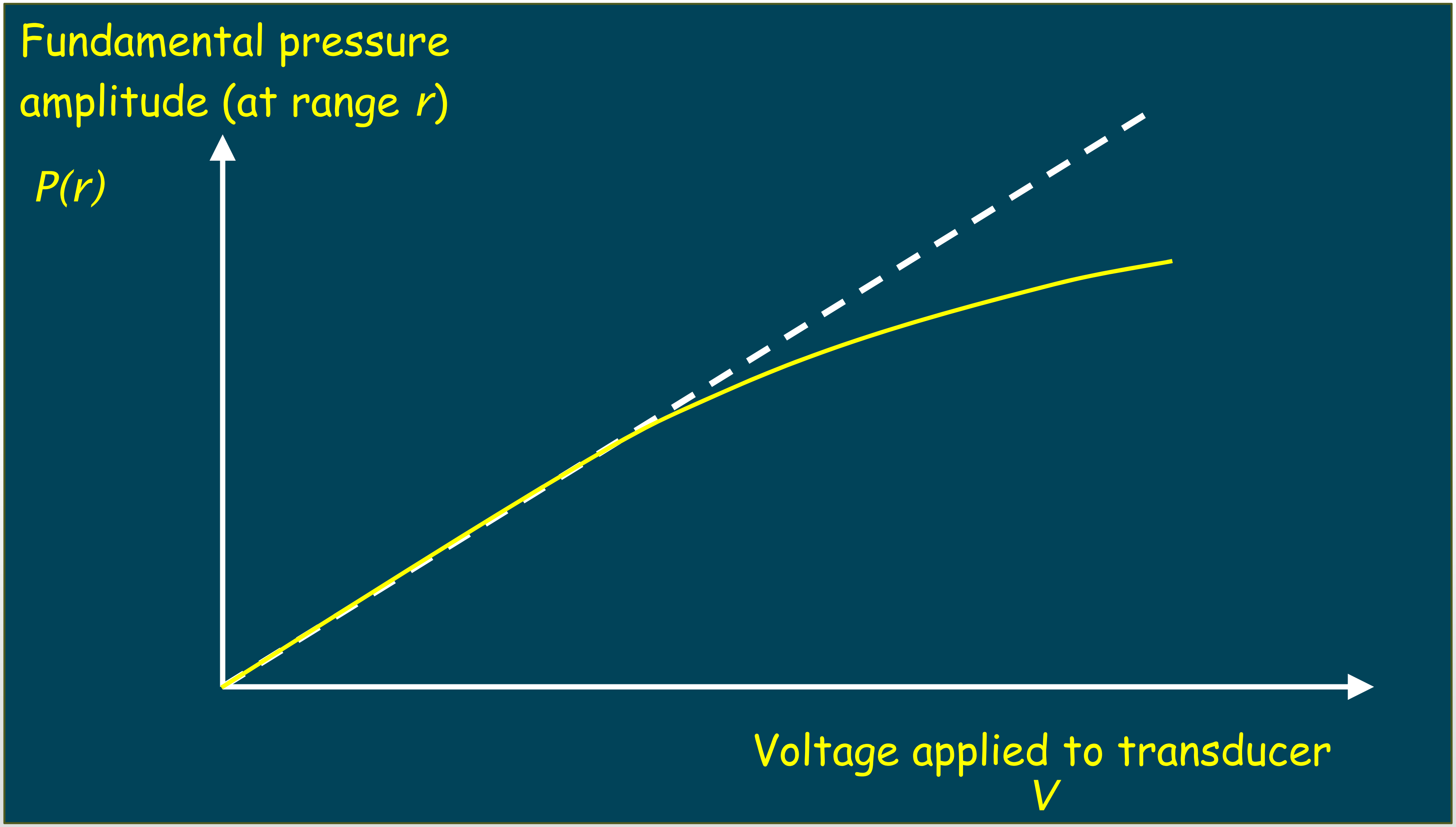
Saturation



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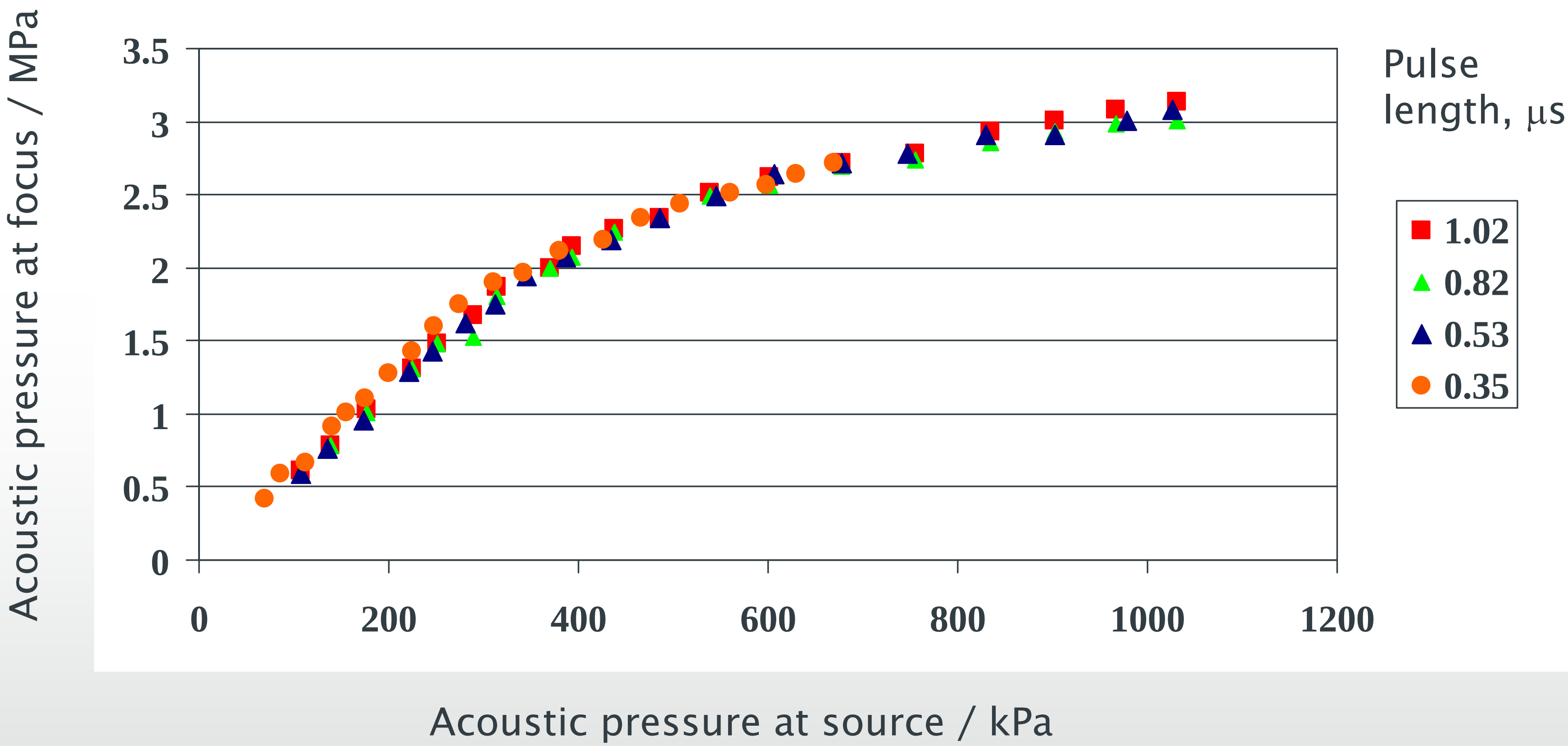
Acoustic Saturation at the Focus



Measured fundamental acoustic pressures at focus for 4 pulse lengths; 3.3 MHz, 19 mm circular source with 95 mm focal length.

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Enhanced Attenuation (Water)

Source Pressure / MPa	Enhanced Attenuation (Theory) / dB cm^{-1}	Enhanced Attenuation (Experiment) / dB cm^{-1}	Enhancement Factor
0.35	1.0	1.1 ± 0.14	19
0.43	1.34	1.25 ± 0.08	24
0.57	1.67	1.3 ± 0.1	31

Results for attenuation in focal region for 5 MHz focused source in water; attenuation calculated by reference to results for a 0.02 MPa source pressure.

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Thank You