MEASUREMENT
OF
TRANSMITTED
ULTRASONIC POWER

AND
DETERMINING THE
IMPEDANCE OF THE LOAD TERMINATING
AN
ULTRASONIC SYSTEM

Nicholas Maropis, February 2008

TAKEN FROM REPORTS ON WORK CARRIED OUT AT AEROPROJECTS
INCORPORATED, WEST CHESTER, PA
1950'S AND 1960'S

FUNDAMENTALS OF
ULTRASONIC WELDING
JONES J. BYRON, NICHOLAS MAROPIS,
JOHN G. THOMAS, DENNISON BANCROFT, CONSULTANT
AND PROFESSOR WILLIAM C. ELMORE, CONSULTANT
BUREAU OF NAVAL WEAPONS (FINAL REPORTS--May 1959),
AND DECEMBER 1960

MEASUREMENT OF ULTRASONIC POWER AND DETERMINING THE IMPEDANCE OF A LOAD TERMINATING AN ULTRASONIC SYSTEM

ABSTRACT

The period of 1959—1965 was unique in that it was relatively easy to obtain support for theoretical and experimental work aimed at identifying the applicability of ultrasonic energy to industrial and medical applications. Support came from both the government agencies and commercial/industrial companies. The work reported here relates to methods for measuring directly the ultrasonic power transmitted by a metallic wave-guide and a method for determining the impedance of a load terminating an ultrasonic system, and comes directly from reports prepared in 1959 and 1960. Both theoretical and experimental results are provided. This work also led to a patent on the methods, "METHOD AND APPARATUS FOR MEASUREMENT OF ACOUSTIC POWER TRANSMISSION AND IMPEDANCE", William C. Elmore, Dennison Bancroft, Nicholas Maropis, and Carmine F. DePrisco—(3,208,241) November 29, 1966.

INTRODUCTION

The purpose of this document is to bring forth theoretical and experimental work which has not been previously published (to our knowledge) in the open literature. We discussed the early work with Mrs. Janet Devine, a member of the UIA Board, and agreed that this is valuable information and should be brought to the attention of the Ultrasonic Industry.
A method for measuring directly, the ultrasonic (U/S) power transmitted by a metal rod or wave-guide is described, and in the second section, a method is described for determining the impedance of a load terminating an ultrasonic system.

The measurement of transmitted U/S power passing any plane along a metal wave -guide was not possible in the 1959-1960 time period, as far as we knew. One could, for example, measure the electrical power applied to transducers, and knowing their characteristics, deduce a level of U/S power to a load. After a number of calibrations, and a knowledge of—system--volts, amps, and phase angle, one could obtain somewhat better values and most relied upon such data.

Early in the development of the U/S metal Welding Process, a principle question was—how much power is required to make any given U/S metal weld? One must understand that in the latte 1950's, the instrumentation and technology of today did not exist. We got by with oscilloscopes and Simpson meters! Furthermore, the old fashion high power electron tube generators were not very efficient. The large output tubes created enough heat that room temperatures were affected. Their overall efficiencies were in the 40% to 50% range.

The transducers for high power were of the thin oxidized and laminated magnetostrictive types. And they too were of relatively low efficiency, generally in the 35% to 40% range.

Thus attempting to determine the actual power transmitted to the "weld zone" was a problem, and especially as welding of thicker metal samples was being achieved. How much power was required, and how thick a metal strip can one weld? One simply plotted data and extrapolated. But one did not know the true power utilized.

Professor William C. Elmore of Swarthmore College, consulting on a regular basis, suggested that we investigate the technology of transmission of micro-wave energy, and determine if an analogous method could be developed for U/S energy, and with his and Professor Dennison Bancroft's (also of Swarthmore College) help the methods described were developed.

Professor Elmore did the initial theoretical work and developed equations associated with the measurement of U/S power, and this was later picked up by Professor Dennison Bancroft, also consulting at the time, who developed the theory for determining the impedance terminating an U/S system. The experimental work was carried out by the staff, including the development for the use of a John Fluke VAW meter for direct read-out of the U/S power being transmitted by a wave-guide.

Professor Bancroft later developed a proof that the VAW meter method did provide accurate measurements. This was providing average values as a consequence of the low frequency response of the meter movement, the signals obtained by epoxy bonding very small piezo crystals to the wave guides were applied directly to storage oscilloscopes and the resulting elliptical patterns —sometimes circles--were photographed and data taken from the photographs.

THEORY

This section is concerned with theory of a method for measuring U/S power associated with longitudinal waves passing along a uniform slender rod. The rod can form part of the coupling system (wave-guide) between the transducer and the load, and its impedance $(A\rho c)$ should preferably match that of the connecting members at each end. It will be shown that power passing from the transducer end to the load end of the rod is

$$\overline{P} = \frac{R_c \omega^2}{2S} \xi_{max}^2 \quad \text{--(1)}$$

Where

$\overline{P}$=the power in watts

$R_c = A\rho c$ in Kg/sec= characteristic waave guide impedance

$\omega = 2\pi f$ in radians /sec

ξ_{max} = maximum amplitude at "loop" position in meters

ξ_{min} = minimum amplitude at a "node" position

S= standing wave ratio- $\frac{\xi_{max}}{\xi_{min}}$

As $\dot{\xi}_{max} = \omega \xi_{max}$ is the maximum particle velocity of the longitudinalwaave in m/sec, we have

$$\overline{P} = \frac{R_c \omega^2 \dot{\xi}^2_{max}}{2S} \text{------------------------------------} (2)$$

The following well known equations hold for a longitudinal wave on a uniform slender rod. (The term slender means that the rod diameter is considerably less than one wavelength.)

$$\frac{\partial^2 \xi}{\partial t^2} = c_2 \frac{\partial^2 \xi}{\partial x^2} \text{------------------------------} (3)$$

$$c = \sqrt{\frac{E}{\rho}} \text{--} (4)$$

$$F = -AE \frac{\partial \xi}{\partial x} \text{--} (5)$$

where ξ= particle displacement (m)

c= wave (longitudinal mode) velocity (m/sec)

E= Young's modulus (newton's/m^2)

ρ= density (kg/m^3)

A= cross sectional area of rod (m^2)

t= time (sec)

x= position along rod (m)

F= instantaneous local force associated with the wave (newtons).

Unless the load exactly matches the (resistive) characteristic impedence of the coupling system, waves are reflected from the load toward the transducer, and a pattern of standing waves is established on the rod. If an origin for x is chosen at a posistion of maximum particle displacement, and the positive direction is from the transducer toward the load, the pattern of standing waves is given by the real part (R.P.) of

$$\xi = \xi_+ e^{j(\omega t-kx)} + \xi_- e^{j(\omega t+kx)} \qquad (6)$$

i.e., R.P. $\xi=\xi_+\cos(\omega t-kx) + \xi_-\cos(\omega t+kx)$, ____________ (6a)

where $k= 2\pi/\lambda= \omega/c$,

ξ_+ is the amplitude of the wave traveling toward the load and

ξ_- is the amplitude of the reflected wave traveling back toward the transducer.

The particle velocity at any point can be found from R.P. of

$$v= \frac{\partial\xi}{\partial t} = j\omega\xi \qquad (7)$$

And-- the force from the R.P. of

$$F= -AE\frac{\partial\xi}{\partial x} = jAEK\,\{\xi_+ e^{j(\omega t-kx)} - \xi_- e^{j(\omega t+kx)}\}. \qquad (8)$$

The instantaneous power passing any point is given by

P= (R.P. of F) x (R.P. of v). ____________________ (9)

As

$$\text{R.P. of } v = -\omega\ \{\xi_+ \sin(\omega t-kx) + \xi_- \sin(\omega t+kx)\},$$

$$P = AEK\ \omega\{\xi_+^{\ 2} \sin^2(\omega t-kx) - \xi_-^{\ 2} \sin^2(\omega t+kx)\}, \quad \text{-----} \ (10)$$

The time average transmitted power, therefore, is

$$\overline{P} = \frac{1}{2}\ AEK\ \omega(\xi_+^{\ 2} - \xi_-^{\ 2}) = \frac{1}{2}[A\rho c]\ \omega(\xi_+^{\ 2} - \xi_-^{\ 2}). \quad \text{-------------} \ (11)$$

The particle amplitudes ξ_+ and ξ_- of the two traveling waves are not directly observable. However Equation (6a) shows that the maximum amplitude (at x= 0, $\frac{\lambda}{2}, \frac{3\lambda}{2}, ...$) of the standing wave is

$$\xi_{max} = \xi_+ + \xi_-$$

Hence equation (11) can be written

$$\overline{P} = \frac{1}{2} R_c \omega^2 \xi_{max}\ \ \xi_{min}, \quad \text{-----------------} \ (12)$$

If one defines the standing-wave ratio

$$S = \frac{\xi_{max}}{\xi_{min}} \quad \text{------------------------------} \quad \text{-----} \ (13)$$

Then we have

$$\overline{P} = \frac{1}{2S} R_c\ \ \omega^2 \xi^2_{max} = \frac{1}{2} R_c S \omega^2 \xi^2_{min}$$

$$\overline{P} = \frac{1}{2S} R_c\ \ \omega^2 \xi^2_{max} = \frac{1}{2} R_c S \omega^2 \xi^2_{min} \ \text{----------------} (14)$$

Equations (12) and (14) show that if one can measure $\xi_{max}\, and\, \xi_{min}$ (or the corresponding Particle velocities $v_{max=\omega\xi_{max}}$, and $v_{min} = \xi_{min}$ by using some type of electrical pick-up (such as very small piezo crystals), responsive at the ultrasonic frequencies), the transmitted power can be determined. These equations are the acoustic analogue of well-known equations for an electrical transmission line—micro-wave technology.

Acoustic Units

In thinking about acoustic transmission-line problems it would appear helpful to make use of the analogy to the electrical case as shown in the following table:

Electrical	**Acoustic Rod**	**Unit**	**Name**
Resistance R	R= Ac	kg/sec	(acoustic) ohm
Current i	Particle velocity v	m/sec	(acoustic) ampere
Voltage V	Force F	Newtons	(acoustic) volt
Charge Q	Displacement	m	(acoustic) coulomb
Inductance L	Mass M	kg	(acoustic) henry
Capacitance C	Compliance	m/Newton	(acoustic) farad

As an example of the order of magnitude of the acoustic values, consider the case of a one-inch-diameter ($A= 5 \text{ x } 10^{-4} m^2$) steel rod passing along (transmitting) 100 watts of power with unity standing-wave ratio. Its characteristic impedance is very nearly ($A=5 \text{ x } 10^{-4} m^2, \ \rho = 7.8 \text{ x } 10^3 kg/m^3, \ C = 5.2 \text{ x } 10^3 m/sec$) $R_c \sim 20{,}000$ kg/sec or ac-ohms.

As $\overline{P} = \frac{1}{2} R_c v^2_{max}$, the peak or maximum particle velocity becomes (the root-mean-square value = $v_{max} / \sqrt{2}$)

$v_{max} = 1/10$ m/sec or ac-ampere.

If the frequency is about 15KHz so that $\omega \cong 10^5$, and the particle amplitude is $\xi_{max} = 10^{-6} m$ or ac-coulomb= 1 micron.

The maximum force in the rod can be computed from $P = \frac{1}{2} F_{max} \text{ x } v_{max}$.

In our example, P=100 watts and v_{max} = 1/10 m/sec, so that F_{max} = 2000 Newtons.

The tensile stress is $F_{max}/A = 2000/5.06 \text{ x } 10^{-4} = 4 \text{ x } 10^6$ newtons/m^2.

The maximum strain can be found from Equation (5), (Y= 20 x 10^{10} newtons/m^2):

$$\left(\frac{\partial \xi}{\partial x}\right)_{max} = \frac{4 \text{ x } 10^6}{2 \text{ x } 10^{11}} = 2 \text{ x } 10^{-5} \qquad (15)$$

Which is about 2 percent of the strain at the yield point, i.e. , 10^{-3} .

Note that when the standing-wave ratio S is greater than unity, which is the usual case, the maximum force, velocity and strain are each increased by a factor ‘S’ at the so-called loop positions for these quantities, and decreased by the factor 1/S at the so-called nodal positions.

Supplementary Information

Attention is called here to the use of a transmission line calculator (P.H. Smith, January 1939 and January 1944 Electronics) for solving problems involving waves on a linear transmission line. The chart simplifies the complicated algebra involved in such problems. In particular, one can compute the impedance presented by a load by measuring (1) the standing-wave ratio (2) the position (in terms of wavelength) of a loop or a node) from the load. One must know the load impedance in order to design a coupling section between transducer and load to bring about a greater transmission of ultrasonic power between the two. Thus a technique of the sort proposed for measuring the transmission of ultrasonic power also gives other useful information about the system.

It is clear from the foregoing analysis that for the purpose of standing-wave-ration measurements, the particle velocities or accelerations can be used to measure power. Thus since for sinusoidal waves, $\ddot{\xi} = -\omega^2 \xi$, one may write

$$\bar{P} = \frac{1}{2\omega^2} A\rho c(\ddot{\xi}_+^{\,2} - \xi_-^{\,2}) \text{ ------------------ } (16)$$

Hence it is not important whether one measures, ξ, $\dot{\xi}$, or $\ddot{\xi}$ or some linear combination of them, though one must clearly recognize what quantity is being measured in order to compute $\bar{P}$. In practice, it suffices to know the proportionality constant between amplitude and emf developed by a sensor at the working frequency, and in what it follows; we shall assume that the actual displacements are in fact converted into proportional sinusoidal electromotive forces.

Determination of ξ_+ and ξ_-

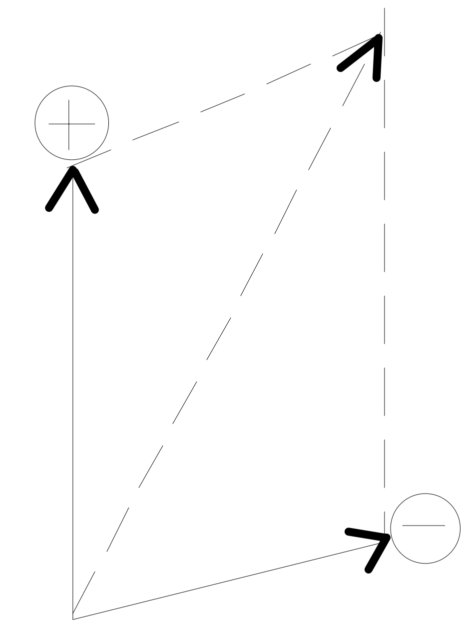
Suppose two pickups are placed on a longitudinal bar, separated by $\frac{1}{4}\lambda$. The displacements, Ξ_1, at the first pickup or sensor, and Ξ_2, at the second pickup, are the real projections of rotating vectors [See Equation (6)],

$$\Xi_1 = \xi_+ \exp j2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \xi_- \exp j2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) \text{ -------------- } (17)^*$$

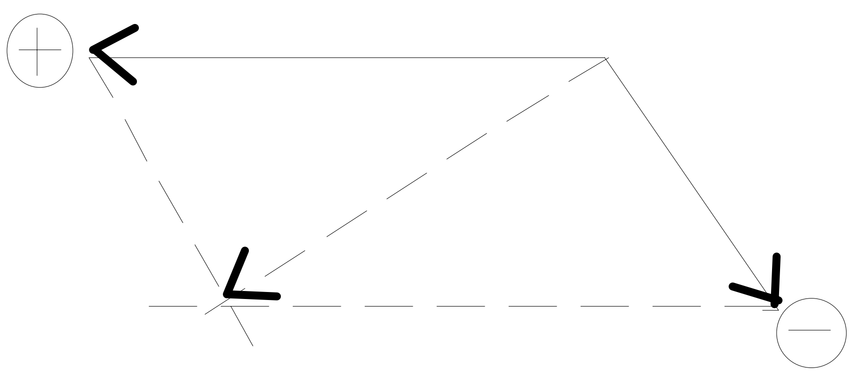
$$\Xi_2 = \xi_+ \exp j2\pi\left(\frac{t}{T} - \frac{x}{\lambda} - \frac{1}{4}\right) + \xi_- \exp j2\pi\left(\frac{t}{T} + \frac{x}{\lambda} + \frac{1}{4}\right) \text{ ------ } (18)$$

------* the symbol exp j( ) throughout denotes $e^{j()}$.

The instantaneous position and composition of such vectors is illustrated in figure A-1 for some instant, say t=0.



a.



b.

The ⊕ and ⊖ symbols refer to the separate terms in equations (3) and (4).

It will be noted that though the diagram in figure A-1(a) was drawn in an arbitrary manner, the corresponding diagram in Figure 1(b) was constructed by rotating the ⊕ vector of Figure A-1(a) 90° counterclockwise, and the ⊖ vector of Figure A-1 (a) 90° clockwise, as required by the relationship of Equation (4) to Equation (3).

A consideration of Figure A-1 immediately suggests how to determine ξ_+ and ξ_-: If one of the diagrams is rotated through 90° [say Fig. A-1 (a) rotated 90°], and the two displacements (which is to say, the corresponding emf's) added, the resultant is 2 ξ_+. Analytically one can perform the same operations with Equations (17) and (18). Thus rotation of

ξ_1 $90^o clockwise yields$

$$\text{-j}\Xi\text{=-j}\xi_+ \exp\ \text{j}2\pi(\frac{t}{T}-\frac{x}{\lambda})-\text{j}\xi_- \exp\ j2\pi(\frac{t}{T}+\frac{x}{\lambda})--------(19)$$

But since exp

$$\text{j}(-\frac{\pi}{2})=\text{j},\ \exp\ \text{j}(+\frac{\pi}{2})=+\text{j, we find}$$

$$\Xi_2=\xi_+\ \exp\ \text{j}2\pi(\frac{\text{t}}{\text{T}}-\frac{x}{\lambda})+\text{j}\xi_-\ \exp\ \text{j}2\pi(\frac{\text{t}}{\text{T}}+\frac{x}{\lambda})-----------(20)$$

$$Hence\ \text{-j}\xi_1+\Xi_2=-2j\xi_+\ \exp\ j2\pi(\frac{t}{T}-\frac{x}{\lambda})--------------(21)$$

and

$$\text{-j}\Xi_1-\Xi_2=-2j\xi_-\ \exp\ j2\pi(\frac{t}{T}+\frac{x}{\lambda})-----------------(22)$$

Hence the amplitude in Equations (20) and (21) are respectively 2 ξ_+ and $2\xi_-$, which if known permit immediate determination of $\overline{\text{P}}$, through Equation (11).

It will be further noted that the phase difference between Equation (21) and Equation (21) is $4\pi\frac{x}{\lambda}$, determination of which serves to locate the maxima and minima of the standing-wave pattern with respect to the points chosen for the pickups. Though the separation of the pickups is critical, the exact location of the pair is immaterial.

Instrumentation

The design of an instrument to effect the necessary measurements is now clear. It involves:

(1) A device for shifting the electrical phase of the output of one pickup by 90°.

(2) A device for subtracting two sinusoidal emf's, and measuring the resultant amplitude.

(3) A device for adding the same two emf's and measuring the resultant amplitude.

(4) A device for measuring the phase difference between the emf's mentioned in items (2) and (3) above.

a. A single instrument capable of performing functions specified by (1) and (2) can be used as a device for performing the function in paragraph (3) simply by interchanging the pickup outputs. However, such a device will not perform the function specified by (4). Again, if two outputs $-\text{j}\Xi_1$ and Ξ_2 are brought into phase, they may be added and subtracted electrically, yielding amplitudes $(\xi_+-\xi_-)$ and $(\xi_++\xi_-)$, the product of which is proportional to $\overline{\text{P}}$.

b. Alternatively, if the raw outputs of the two pickups be used to produce horizontal and vertical deflections respectively on an oscilloscope, the area of the resulting ellipse is proportional to the power transmitted along the bar.

----Proof for paragraph (b) above:

Let the x deflection be the real part of equation (17), and the y deflection be the real part of Equation (4):

$$\left.\begin{aligned} x &= \xi_+ \cos 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)+\xi_- \cos 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right) \\ y &= \xi_+ \sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)-\xi_- \sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right) \end{aligned}\right\} \text{\_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_} \quad (23)$$

Let $2\pi\frac{t}{T}=\theta$, and let $2\pi\frac{x}{\lambda}=\varphi$. These equations may be written

$$\left.\begin{aligned} x &= \xi_+(\cos\theta\cos\varphi + \sin\theta\sin\varphi) + \xi_-(\cos\theta\cos\varphi - \sin\theta\sin\varphi) \\ y &= \xi_+(\sin\theta\cos\varphi-\cos\theta\sin\varphi)-\xi_-(\sin\theta\cos\varphi+\cos\theta\sin\varphi) \end{aligned}\right\} \text{\_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_} \quad (24)$$

The area of the ellipse is of coarse A= $\int y dx$, where the integration covers one complete cycle. Now, from equation (24),

$$dx = \xi_+(-\sin\theta\cos\varphi + \cos\theta\sin\varphi)\, d\theta+\xi_-(-\sin\theta\cos\varphi - \cos\theta\sin\varphi)\, d\theta. \text{\_ \_ \_ \_ \_ \_ \_\_\_} \quad (25)$$

Clearly the required integral, taken with respect to θ instead of x, contains terms in $\sin^2\theta,\ \cos^2\theta,$ and $\sin\theta\cos\theta$. Since the integral for one complete cycle of terms of the last type is zero, we shall ignore them. Accordingly,

$$A=\int_\circ^{2\pi} d\theta \sin^2\theta[-\xi_-{}^2\cos^2\varphi+\xi_+\xi_-\cos^2\varphi-\xi_+\xi_-\cos^2\varphi+\xi_-{}^2\cos^2\varphi]+$$

$$\int_0^{2\pi} d\theta\cos^2\theta[-\xi_+^2\sin^2\varphi-\xi_+\xi_-\sin^2\varphi+\xi_+\xi_-\sin^2\varphi+\xi_-^2\sin^2\varphi].$$

Hence finally, since $\sin^2\varphi + \cos^2\varphi = 1$, and since $\int_0^{2\pi} \cos^2\theta \, d\theta = \pi$, one finds that

$$A = \pi[\xi_-^2 - \xi_+^2].\text{---}(27)$$

Comparison of equation (27) with equation (11) completes the proof. The sign of (A) in equation (27) depends on the direction in which power is being transmitted, which could not be determined.

<u>DETERMINATION OF THE IMPEDANCE TERMINATING AN ULTRASONIC SYSTEM</u>

Determining the load impedance terminating high power ultrasonic systems is important because one can easily damage and in fact destroy transducers. Perhaps one can never match exactly all components from the transducers to the load, but one can achieve conditions for delivery of very high levels of ultrasonic power—of the order of 1000 watts per square inch of wave guide cross section area.
For example, for high power applications such as metal tube drawing, extrusion, and the like—wave guide heating in materials recommended for such service, an upper limit of 6000 watts per square inch of cross section is specified. And if the wave guides are long, such as 130 feet, the max power generally can not exceed 5000 watts per square inch of wave guide cross section.

Professor Bancroft developed the following theory—

A method for determining the transducer load impedance utilizes a knowledge of the characteristic impedance of the transmitting rod (wave guide) and the instrumentation previously described, for the standing wave ratio and the distance from the end of the coupler (wave guide) to the nearest maximum in the standing wave pattern, the terminal impedance can be defined through the equations:

$$\frac{Z_t}{Z_c} = \frac{\xi_{\min} \cos\frac{2\pi}{\lambda} X_0 - j\sin\frac{2\pi}{\lambda} X_0}{\xi_{\max} \cos\frac{2\pi}{\lambda} X_0 - j\xi_{\min} \sin\frac{2\pi}{\lambda} X_0} \text{------------------}(28)$$

Where the notation for this and the following equations is:

Zt= the acoustic impedance terminating the waveguide
Zc = acoustic impedance which characterizes the transmitting or coupling bar.
X0 =distance from the termination to the nearest particle displacement maximum,
And all other symbols are as before.

Acoustic (ie. Ultrasonic) power is transmitted practically, by longitudinal, radial, tensional, transverse, or other well known oscillatory type vibrations. The following theory is developed for deriving equation 28 is developed specifically for longitudinal vibrations; however, it is sufficiently general to permit application to any vibrational mode provided the measurements are related to the appropriate particle motions.

As before, the following equations hold for a longitudinal wave on a slender uniform slender rod of less than a quarter wave length diameter. Then equation 3 and those that follow hold.

It is assumed that a source of acoustic power is delivering energy in the form of wave motion. Unless the load exactly matches the characteristic impedance of the coupling system $(A\rho c)$, waves are reflected from the load toward the transducer, and a pattern of standing waves is established on the rod. If an origin for X is chosen at a position of maximum particle displacement, and the positive direction is from the transducer toward the load, the pattern of standing waves is given as before by

$$\xi = \xi_+ e^{j(\omega t - kx)} + \xi_- e^{j(\omega t + kx)}$$

Where the actual displacement is equal to the real portion of the equation.

Equations 6-12 define the particle velocity $v = \dot{\xi} = \frac{\partial \xi}{\partial t}$, strain $\frac{\partial \xi}{\partial x}$, and force F, where

$$F = -AE\frac{\partial \xi}{\partial x} = jAEK\,\{\xi_+ e^{j(\omega t - kx)} - \xi_- e^{j(\omega t + kx)}\}. \text{ - - - - - - - - - - - - - - - (28)}$$

The impedance across a section of the rod specified by the coordinate X is the ratio of force to velocity. Thus, if Zc represents the impedance of a rod extending indefinitely to the right, so that reflected waves are never observed, is zero and it is easily seen that

$$Zc = \frac{F}{\dot{\xi}} = \frac{kAE}{\omega} = A\rho c \text{ - (29)}$$

For this analysis the coordinate X=0 was chosen to correspond to the position of maximum particle displacement, ξ_{max} in the standing wave. With this hitherto arbitrary origin chosen to correspond to the maximum nearest terminal impedance, the distance from X=0 to the terminal surface will be designated $X_\circ$. Then at this coordinate is found

$$Z_t = \frac{F}{\xi} = \frac{EAk}{\omega}\;\frac{\xi_+ e^{j(\omega t - kX)} + \xi_- e^{j(\omega t + kX)}}{\xi_+ e^{j(\omega t - kX)} - \xi_- e^{j(\omega t + kX)}} \text{ - (30)}$$

$$Z_t = \frac{EAk}{\omega}\;\frac{(\xi_+ - \xi_-)\cos kX_\circ - j(\xi_+ + \xi_-)\sin kX_\circ}{(\xi_+ + \xi_-)\cos kX_\circ - j(\xi_+ - \xi_-)\sin kX_\circ} \text{ - - - - - - - - - - - - - - (31)}$$

Examination of equation 5 reveals that the maximum and minimum particle displacements. ξ Respectively

$$\xi_{max} = \xi_{+} + \xi_{-}$$

$$\xi_{min} = \xi_{+} - \xi_{-} \quad \text{------------------------------} \ (32)$$

Then combining Equation (31) and (32) gives

$$Z_t = \frac{EAk}{\omega} \left[\frac{\xi_{min} \cos kX_{\circ} - j\, \xi_{max} \sin kX_{\circ}}{\xi_{max} \cos kX_{\circ} - j\, \xi_{min} \sin kX_{\circ}} \right] = \text{-------------------} (33)$$

$$\frac{Z_t}{Z_c} = \frac{\xi_{min} \cos \frac{2\pi}{\lambda} X_{\circ} - j\, \xi_{max} \sin \frac{2\pi}{\lambda} X_{\circ}}{\xi_{max} \cos \frac{2\pi}{\lambda} X_{\circ} - j\xi_{min} \sin \frac{2\pi}{\lambda} X_{\circ}} \text{-----------------------}(34)$$

We now have three equations for determining the U/S power being transmitted by any given waveguide, and for determining the impedance of the load terminating an U/S system, and in both cases utilize the standing wave ratio associated with the transmission of U/S power. These are:

Equation 13, the standing wave radio $S = \frac{\xi_{max}}{\xi_{min}}$

Equation 14, the power equation $\bar{P} = \frac{1}{2S} R_c \ \omega^2 \xi_{max}^2 = \frac{1}{2} R_c S \omega^2 \xi_{min}^2$,and

Equation 34, the impedance equation $\frac{Z_t}{Z_c} = \frac{\xi_{min} \cos \frac{2\pi}{\lambda} X_0 - j\xi_{max} \sin \frac{2\pi}{\lambda} X_0}{\xi_{max} \cos \frac{2\pi}{\lambda} X_0 - j\xi_{min} \sin \frac{2\pi}{\lambda} X_0}$

Note that the R.P. of this equation is

$$\frac{Z_t}{Z_c} = \frac{1}{S}, \text{ and}$$

$$Z_t = \frac{Z_c}{S} \quad \text{------------------------------------(35)}$$

so that the impedance terminating U/S system relative to the impedance of the wave guide, varies inversely as the standing wave ratio which can be measured, and with the wave guide impedance known, one can readily calculate The resistive component of Z_t which is the value we seek.

Ancillary Information

The piezo crystals used to develop data as discussed later, were approximately 3/8"L x 1/8"W x 0.025" thick. These were sandwiched between two pieces of brass strip, and bonded with low temperature lead--tin alloy solder.

These were then carefully epoxy bonded to milled flat sections on the wave guide, and one quarter wave length apart. The width section was oriented parallel to the axis of the wave guide. These were then carefully taped together with the connecting wires to prevent damage and possible extraneous signals from anything loose.

These were calibrated by coupling the instrumented wave guide to an "Acoustic Absorber" which could accept up to 5 KW of vibratory power, and using calorimetric methods establish emf values from each crystal---essentially adjusting the oscilloscope x-y gains for a circular pattern, and whose area was proportional to the U/S power delivered to the load.

In practice, this instrumented wave guide provided accurate values for both “S” and “P”, and a good estimate for the impedance of the load.
For example, initial results of U/S applied to wire drawing indicated that a high vibratory amplitude yielded the best results. This was a transducer to die amplitude gain of
5 to 7 x.

However, in U/S Tube Drawing with the U/S power applied through the ID sizing plug, a lower SWR was associated with lower gain factor—of the order of 2.5 to 3.0.

APPLICATION OF THE KEY EQUATIONS

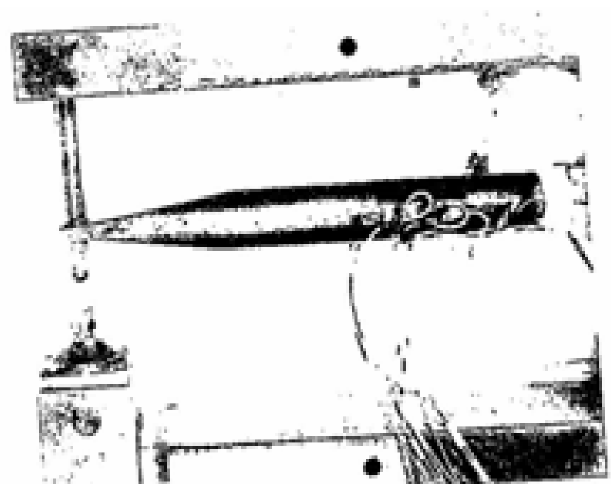
The data that follow were developed by the staff at Aeroprojects Incorporated in 159 and 1960 and were applied primarily to U/S Metal welding. Application to U/S metal working processes was just beginning.

The following figures show the various SWR forms and values under the noted conditions, and provide insight into the potential of this technique.

Figure 1 shows a laboratory wedge-reed system of a 2.0KW ultrasonic metal welding system. The cylindrical portion of the coupling system—between the transducer and the wedge, serves as the wave guide and is instrumented as described earlier.

Figure 1

U/S Welding System



EXPERIMENTAL WELDING ARRAY WITH SWR SENSING ELEMENTS ATTACHED TO ULTRA-SONIC COUPLER

Figure 2 shows oscillograms of the wave forms obtained from an instrumented wave guide that was powered with no load—air load and an SWR of About 16.5.

The end of the wave guide was then put into water, and it can be seen that the SWR decreased a small amount, and finally the waveguide was attached to a tapered horn that was lead-tin soldered into a cylinder of pure lead.
The SWR went down to a 1.3—1.5 range which indicates that the system is delivering power to the load.

Note on these pictures that the axes of the ellipses are rotated from the 'Y' axis which indicates a phase shift.

The transducer was of the nickel laminate type and required phase correction. We do not recall if such was done, although we generally made such corrections when setting up the systems, so we presume that this is due in part at least to some reactive component in the load impedance.

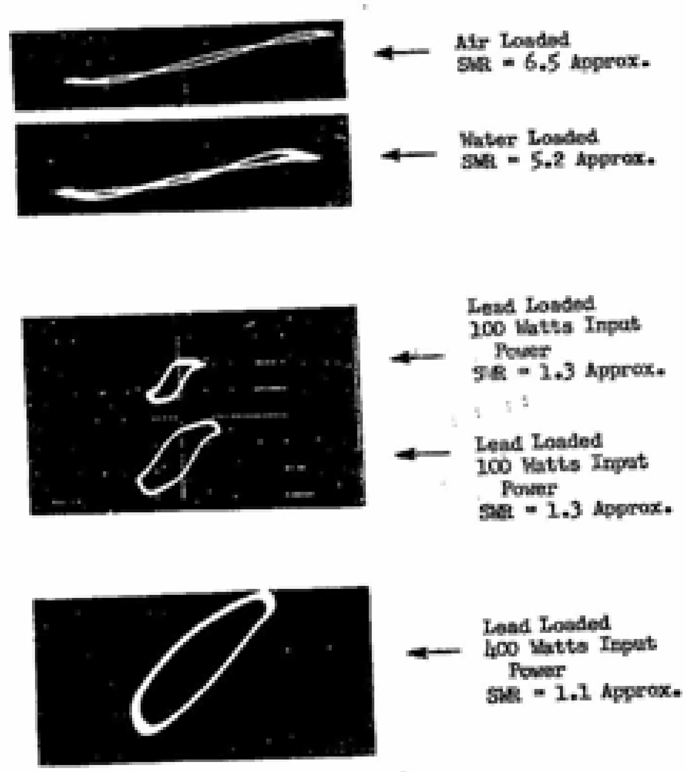


Figure 2

SWR For Three Load Conditions

OSCILLOGRAMS SHOWING SEVERAL COUPLER LOADING CONDITIONS

Figure 3 is interesting in that the SWR forms and values reflect the conditions at initiation of the weld cycle, and after coupling of the energy to form the weld has occurred.

Those of you familiar with U/S metal welding—and I think this must happen to some extent in U/S plastic welding as well—know that there is a time lag between initiation of the weld cycle and coupling when the weld is actually being formed. A number of variations of the settings for U/S welding have been made to control and/or reduce this lag time and improve welding efficiency, and some of these apparently do help. These include programmed clamp force, frequency wobble (slight cyclic variation of frequency during initiation of the weld cycle, and programmed power variations. We tried these as well and sometimes we saw some positive changes and sometimes we did not.

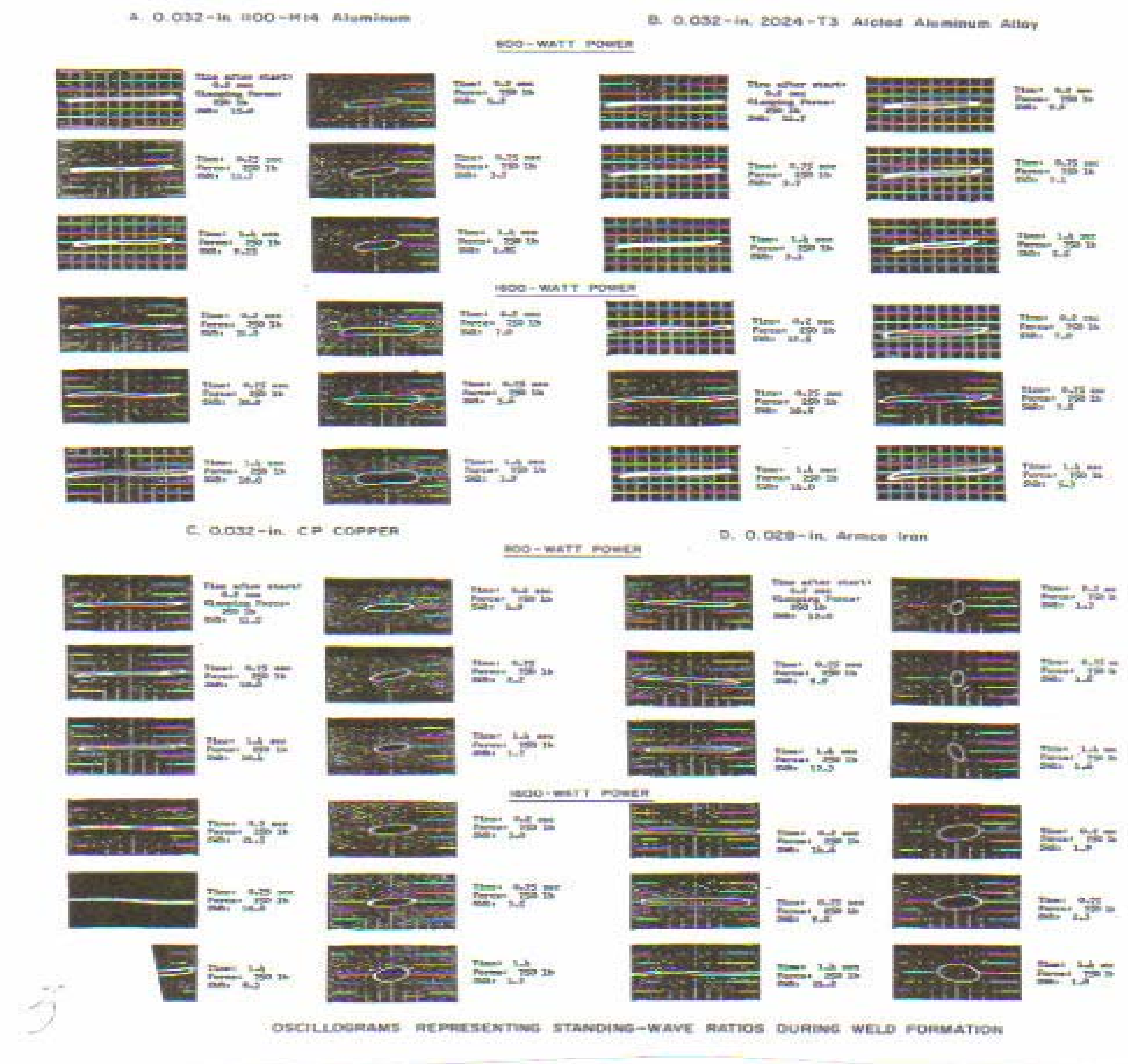


Figure 3: Oscillogram SWR Traces At Initiation Of Weld And Near End Of Cycle

Table 1 provides a summary of data developed from the photographs of figure 3.

TABLE I

RESULTS OF STANDING-WAVE-RATIO MEASUREMENTS DURING WELDING OF SEVERAL MATERIALS

Electric Power to Transducer (watts)	Clamping Force (lb)	Time After Start of Weld (sec)	Standing-Wave Ratio	Ellipse Area (in²)	Acoustic Power (watts)	Efficiency (%)
		A. 0.032-Inch 1100-H14 Aluminum				
800	250	0.2	15.0	0.030	40	5.0
		0.75	11.7	0.076	100	12.5
		1.4	9.75	0.087	115	14.4
	750	0.2	6.2	0.110	145	18.1
		0.75	5.3	0.110	145	18.1
		1.4	2.35	0.100	130	16.2
1600	250	0.2	31	0.071	90	5.6
		0.75	30	0.081	105	6.6
		1.1	16	0.150	195	12.2
	750	0.2	7.0	0.330	430	26.8
		0.75	5.5	0.210	275	17.9
		1.4	3.9	0.190	250	15.6
		B. 0.032-Inch 2024-T3 Alclad Aluminum Alloy				
800	250	0.2	11.7	0.079	95	11.9
		0.75	9.7	0.060	80	10.0
		1.4	9.1	0.060	80	10.0
	750	0.2	9.8	0.116	150	18.8
		0.75	7.1	0.086	110	13.7
		1.4	5.5	0.076	100	12.5
1600	250	0.2	17.5	0.180	155	9.7
		0.75	15.5	0.130	170	10.5
		1.4	14.0	0.070	90	5.6
	750	0.2	7.0	0.200	260	16.2
		0.75	7.8	0.200	260	16.2
		1.4	9.3	0.200	260	16.2
		C. 0.032-Inch C.P. Copper				
800	250	0.2	11.5	0.080	105	13.1
		0.75	10.8	0.066	85	10.6
		1.4	10.4	0.086	110	13.7
	750	0.2	4.9	0.120	155	19.4
		0.75	2.10	0.066	85	10.6
		1.4	1.7	0.066	85	10.6
1600	250	0.2	21.5	0.075	100	6.3
		0.75	16.0	0.066	85	5.3
		1.4	8.3	0.160	210	13.1
	750	0.2	3.0	0.120	155	9.7
		0.75	3.5	0.100	130	8.1
		1.4	1.71	0.125	165	10.3
		D. 0.028-Inch Armco Iron				
800	250	0.2	12.0	0.076	100	12.5
		0.75	9.9	0.103	135	16.4
		1.4	12.3	0.126	165	20.6
	750	0.2	1.33R	0.081	105	13.0
		0.75	1.70R	0.050	65	8.0
		1.4	1.65R	0.060	80	10.0
1600	250	0.2	14.6	0.160	210	13.1
		0.75	9.2	0.230	300	18.8
		1.4	21.2	0.081	105	6.6
	750	0.2	1.9	0.150	195	12.2
		0.75	2.3	0.166	215	13.4
		1.4	1.78	0.151	195	12.2

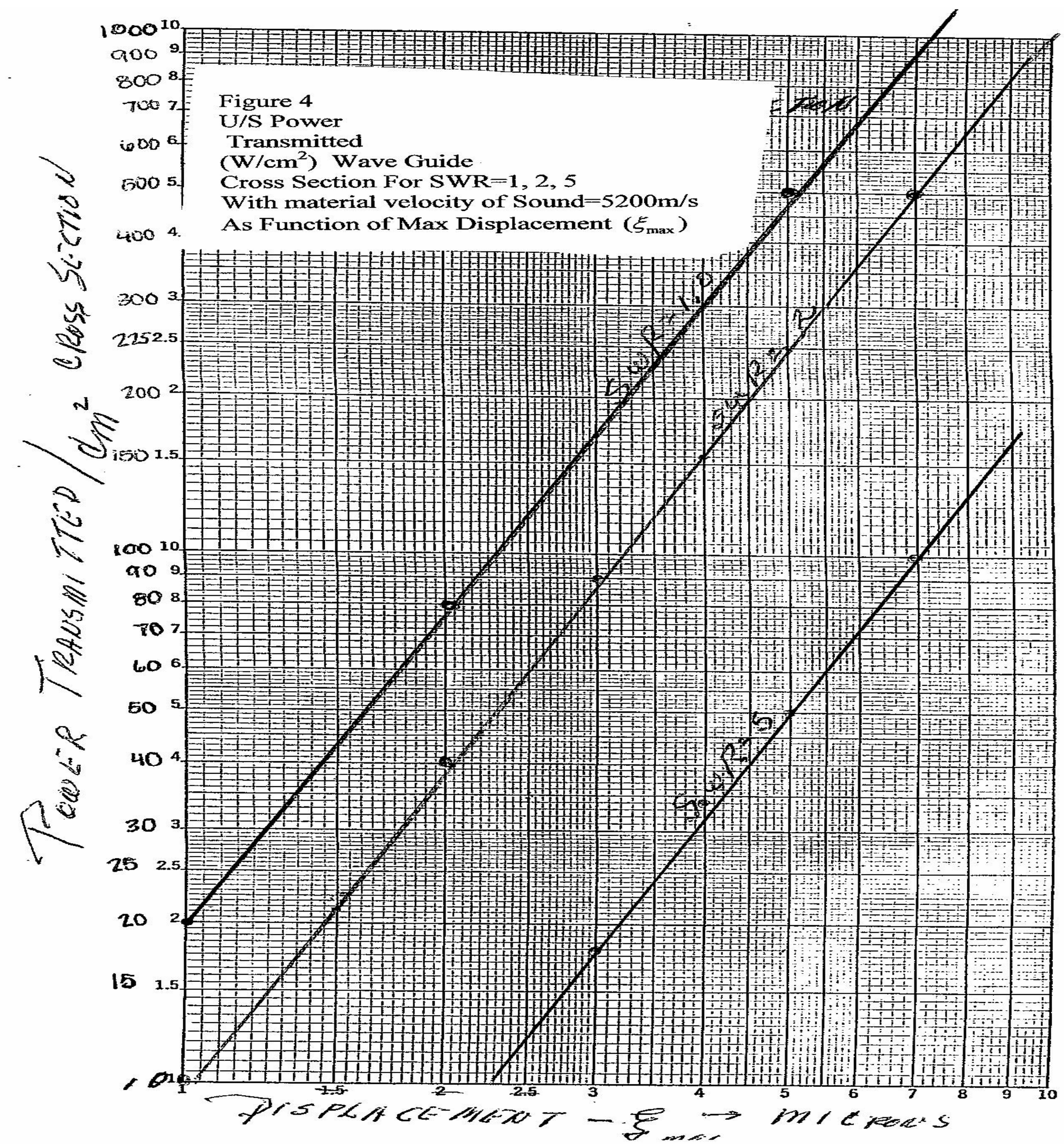
R Indicates that maximum voltage from sensing elements has reversed.

Figure 4 presents a graph of power delivered per (cm)2 of wave guide cross section for power being delivered with a___

SWR=1.0

SWR=2.0

SWR= 5.0



This chart is based on a wave guide characterized by a velocity of sound of 5200 m/sand the equations developed earlier in this presentation.

At an SWR of 1.0, and at a displacement amplitude of about 7.3 microns, one would be transmitting 1000 W/ cm^2 of wave guide cross sectional area..

Experience with high power U/S Tube drawing and extrusion systems indicate that the maximum U/S power that can be transmitted before wave guide heating and losses are encountered is about 6000 W/in2 , or 930 W/cm2. We found these values to hold for alloys such as AISI 303/304, Monel K-500, and Inconel 600. Most alloy steel alloys tend to begin heating at about 600 W/ cm2.

We have observed the SWR on a Back Support Rod (wave guide) during U/S tube drawing and noted that the SWR varies as the length of the undrawn tube changes during the draw. It varies fro about 1.3/1.5 to as much as 3.2/3.6. This variation does not effect the reduction in draw force as it remains relatively constant during the draw cycle for any given applied power level.

Table 2 provides density, velocity of sound, and heat treat conditions for a number of common alloys used in U/S systems for anyone interested.

PHYSICAL PROPERTIES OF SOME MATERIALS USED IN ULTRASONIC SYSTEMS*

METAL	DENSITY* g/cm^3	VELOSITY OF SOUND** M/s
****	****	****
Aluminum		
1100 A	**2.71**	**5200**
Aluminum Bronze	**7.58**	**3700**
Beryllium Copper A	**8.23**	**3750**
Beryllium Copper HT	**8.23**	**3900**
Monel K-500 A	**8.46**	**4480**
Monel R A	**8.84**	**4800**
NiBrAl A	**7.50**	**4600**
Titanium 6al4V	**4.43**	**5075**
Steel 4340 A	**7.83**	**5200**
AISI 303 A	**7.90**	**5030 ± 50**
AISI 304 A	**7.90**	**5030 ± 50**

***Source – Manufacturer's Literature and American Physical Society Handbook**
****Source – Measurements at Aeroprojects Incorporated and since 1971 by NM**

The velocity of sound characterizes materials most importantly when designing high power systems.

A number of years ago Ernie Neppiras of Mullard Ltd in England devised an interesting experimenter in which he measured the peak dynamic strain in a dumb bell type test specimen, and using calorimetric methods measured the amount of heat generated as a function of strain levels. Some years later Mr. Neppiras came to the US, spent some time at the University of Houston, and later worked in the Aeroprojects Laboratory for about a month. He enjoyed the time with us, and we enjoyed discussions with him!!

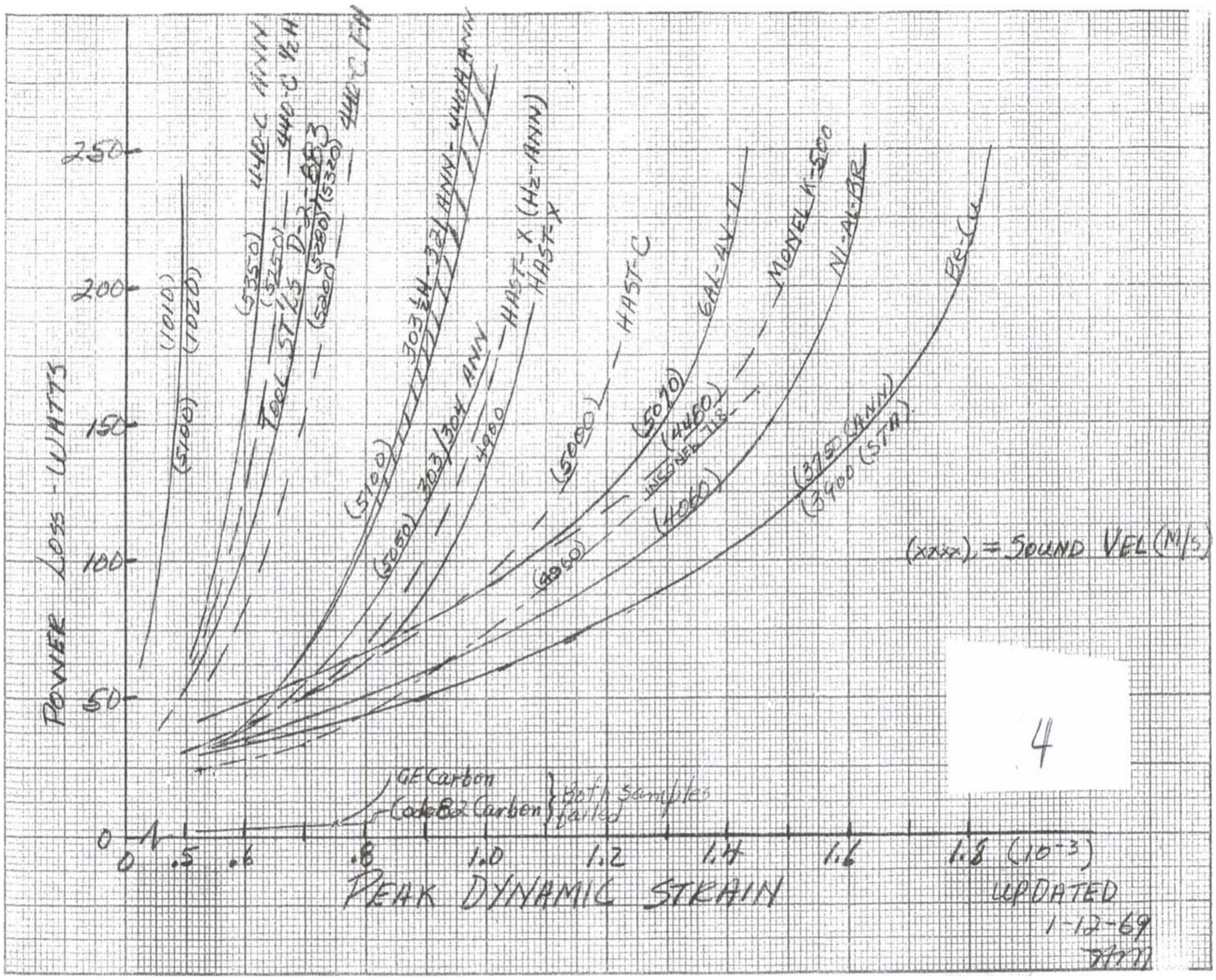
We had reproduced his earlier data and expanded the alloys investigated to include those of interest to Aeroprojects.

A copy of these data plus others since added to the list are shown in figure 5. The original Aeroprojects work was published in an U S Air Force sponsored program final report in the mid 1960's.

We are continuing to design and build U/S drawing systems and fall often fall back on this old work.

It has been a great journey—and once U/S gets into your blood, one can not just break away—it is always calling.

Figure 5


POWER LOSS - WATTS
PEAK DYNAMIC STRAIN
(10^-3)
(xxxx) = SOUND VEL (M/S)
HAST-C
GAL-AV-TI
MONEL K-500
NI-AL-BR
Be-Cu
GE Carbon
Code Bx Carbon
Both samples failed
UPDATED
1-12-69
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