

# Acoustic radiation force creep and shear wave dispersion method for elasticity imaging

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# Mayo Clinic Ultrasound Laboratory Overview



## • Research Areas

- Shearwave Dispersion Ultrasound Vibrometry (SDUV)
- Vibro-acoustography
- Ultrasound imaging

## • Mayo Clinic

- Rich history of clinical collaboration
- Diverse patient population for translation of research techniques.



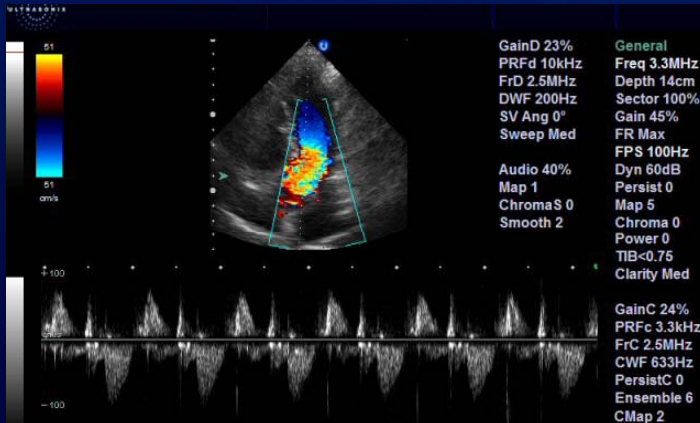
# Medical Imaging Modalities



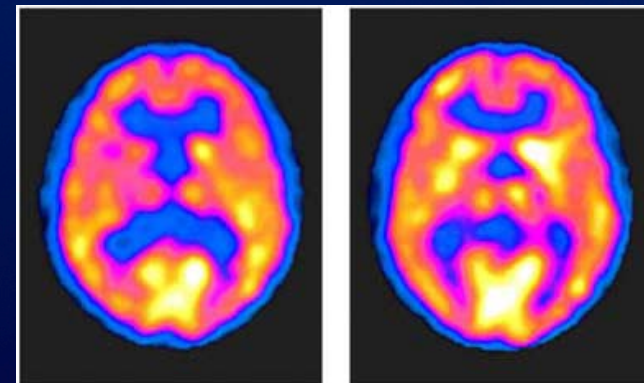
X-ray Computed Tomography  
Contrast: Mass density



Magnetic Resonance Imaging  
Contrast: Proton Density,  
Relaxation Times

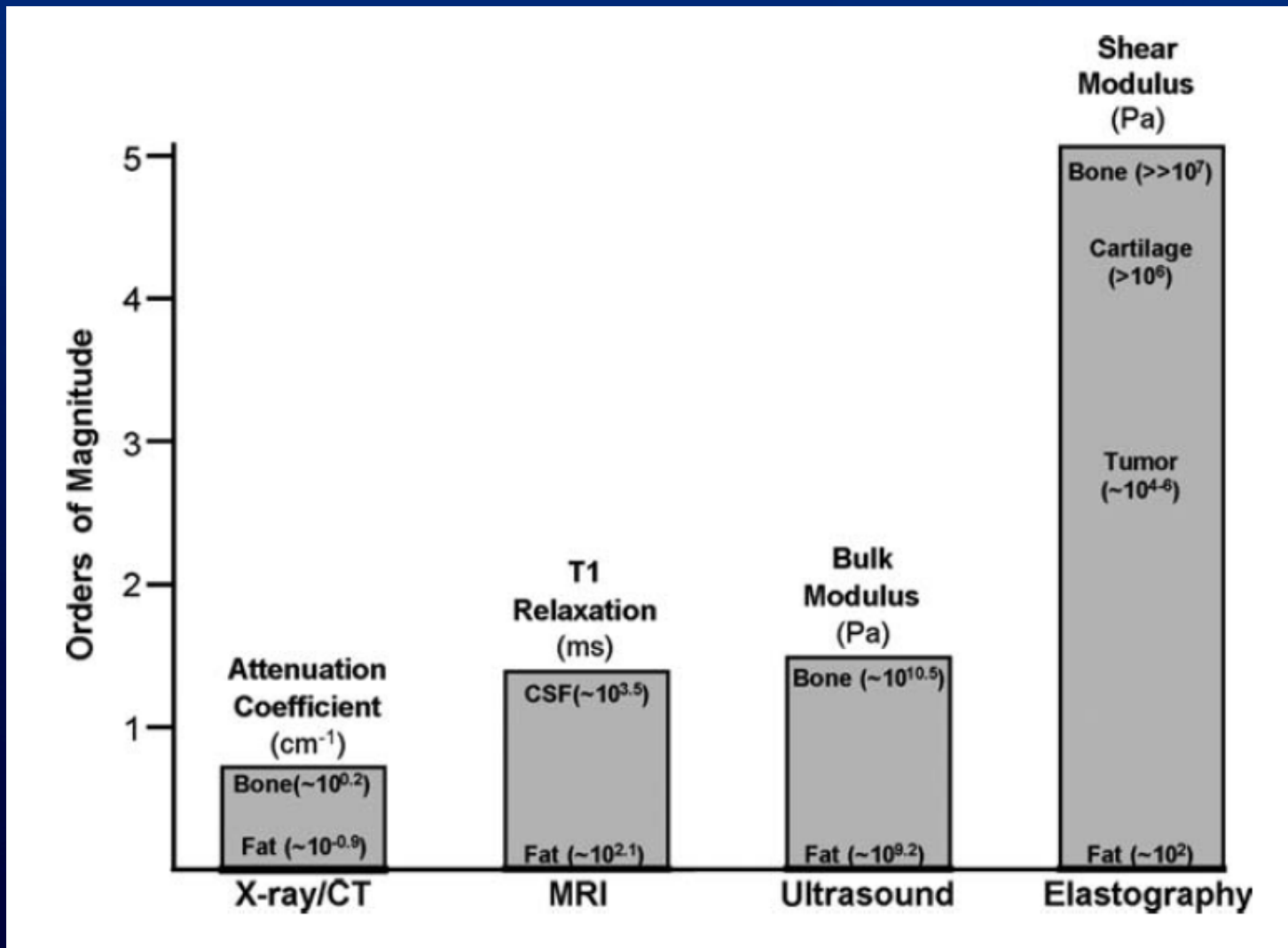


Ultrasound Imaging  
Contrast: Bulk Modulus



PET/SPECT  
Contrast: Radioactive Decay

# Medical Imaging Modalities



# Palpation and its Role in Medicine

- Palpation is fundamental to the practice of medicine.
- The premise of palpation is that diseased tissue “feels” different than normal surrounding tissue, typically the diseased tissue is stiffer.
- Studies have shown a positive correlation between pathology and stiffer tissue in the breast, prostate, liver, and arteries.

# Palpation and Elasticity Imaging

- There are some limitations of palpation:
  - Subjective
  - Dependent on proficiency of examiner
  - Non-reproducible
  - Not sensitive to small or deep lesions
- The goal of any elasticity imaging modality therefore is to produce images that are:
  - Quantitative
  - Reproducible
  - High resolution
  - Noninvasive

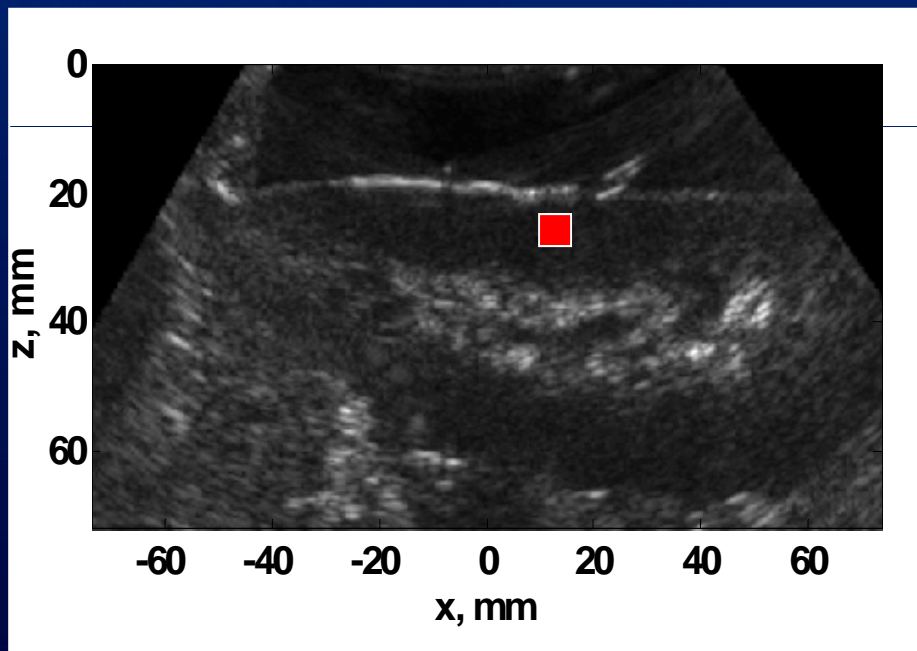


## Shear wave elasticity imaging

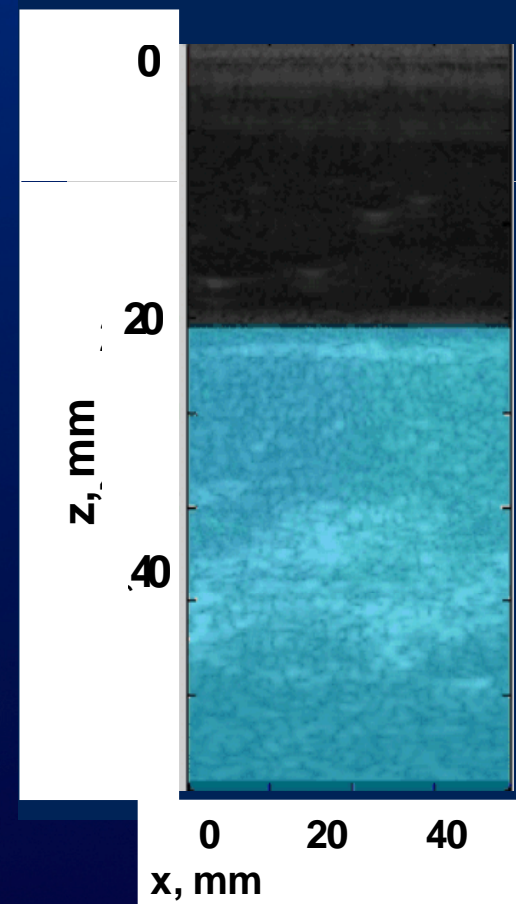
- Introduce shear wave to tissue (external mechanical actuator, ultrasound radiation force)
- Measure shear wave speed with conventional imaging methods (MRI, Ultrasound)
- Shear wave speed depends only on mechanical properties of tissue
- Mechanical properties are estimated by assuming mechanical models (elastic models, viscoelastic models)

# Shear wave elasticity imaging

- Measurements are local (usually 5 – 10 mm<sup>2</sup> regions of interest)



B-scan image of the kidney with Verasonics ultrasound system equipped with linear curved array transducer.



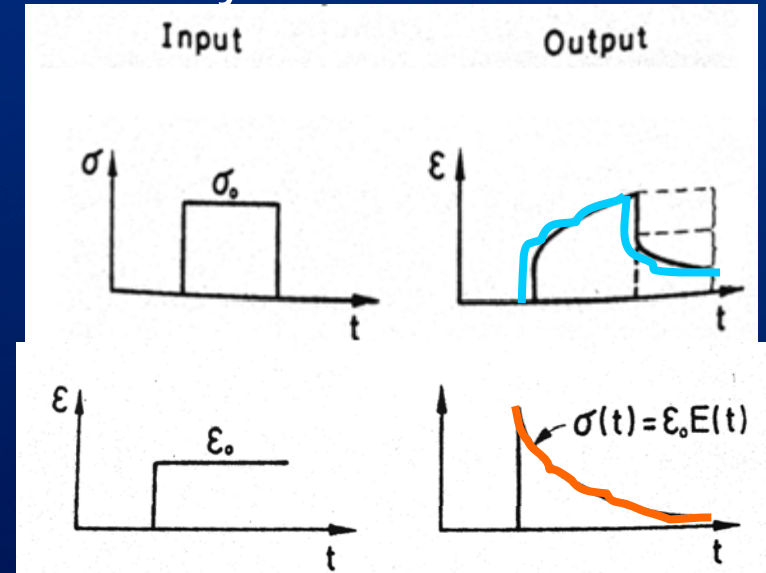


# General background

- Viscoelastic behavior is usually studied by:

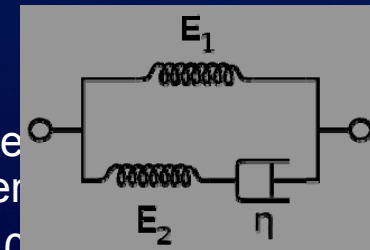
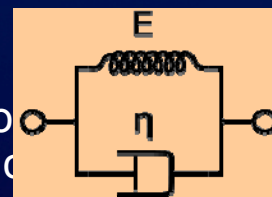
## 1. Static tests

- Creep test
  - Strain under step - stress
- Stress relaxation test
  - Stress under step - strain



- To quantify the viscoelastic properties, a model is usually fit to the data

- Limitations:



- Usually, the first reliable time-point is around 10 seconds
- There are some materials where their viscoelastic behavior occurs early in time (< 10 seconds)

Maxwell

Voigt

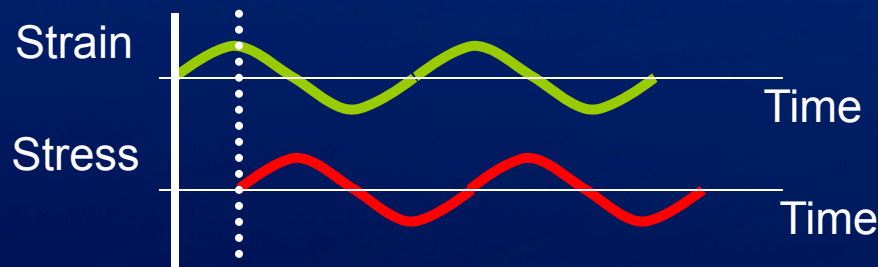
Linear Solid

# General background

Viscoelastic behavior is usually studied by:

## 2. Dynamic tests

- Oscillatory stress/strain applied
- For a sinusoidal strain in time, the stress response in also sinusoidal with a phase shift ( $\delta$ )



$$\varepsilon(t) = \sin(\omega t) \approx \exp(i\omega t)$$

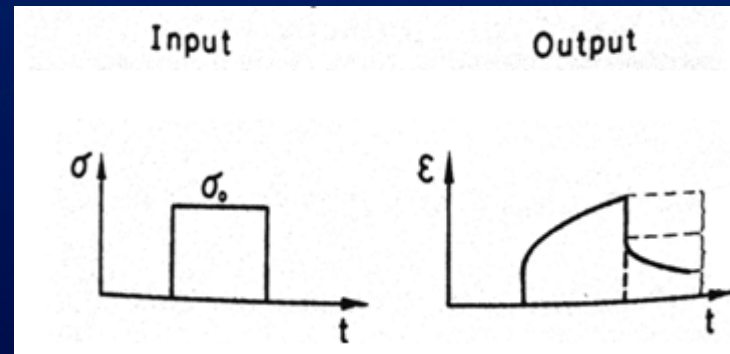
$$\sigma(t) = \sin(\omega t + \delta) \approx \exp(i(\omega t + \delta))$$

$$G^* = \frac{\sigma}{\varepsilon} = \exp(i\delta)$$

- The dynamic modulus,  $G^*$ , is a function of frequency and it is a complex variable  $\rightarrow G^*(\omega) = G_s(\omega) + iG_l(\omega)$ 
  - $G_s(\omega)$  is the elastic or storage modulus
  - $G_l(\omega)$  is the viscous or loss modulus
  - The ratio of  $G_l(\omega)$  to  $G_s(\omega)$  is the loss tangent or  $\tan(\delta)$
- Capable of studying viscoelastic response between  $10^{-8}$  to  $10^3$  seconds
  - Limitations:
    - Measure one frequency at a time
    - Specialized instruments and techniques

## Time vs. frequency measurements

- Creep test → static test to measure viscoelastic behavior
  - Time domain
  - Study viscoelastic behavior from 10 seconds to 'days'
  - Requires a viscoelastic model (Kelvin-Voigt, Maxwell, etc..)



- Creep test will be ideal if
  - The output is converted to frequency domain (complex modulus)
  - No model is required
  - The material creep response is measured early in time



## Complex modulus related to time-creep compliance

- Definition: creep compliance,  $J$ , is the ratio of strain and stress in a creep test.
- The complex modulus,  $G^*(\omega)$ , is related to the complex creep compliance,  $J(\omega)$ , by a convolution<sup>1</sup>

$$\varepsilon(t) = \int_0^t J(t-\xi) \frac{\partial \sigma(\xi)}{\partial \xi} d\xi \quad \longleftrightarrow \quad \text{Fourier Transform (FT)} \quad \therefore G^*(\omega) = \frac{1}{(i\omega) FT[J(t)]}$$

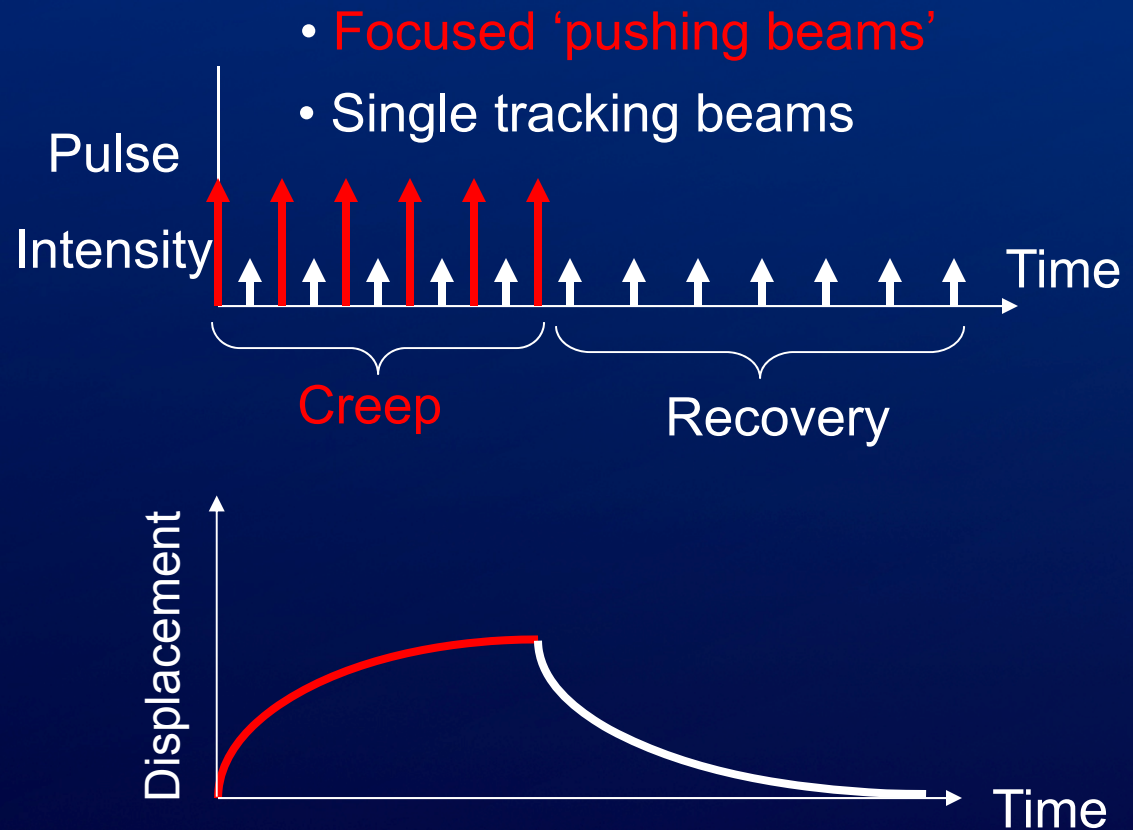
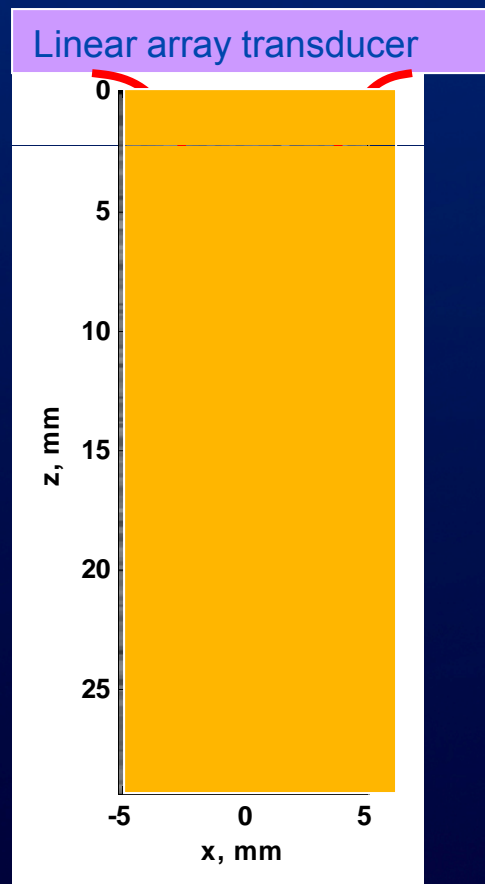
- The problem is that  $FT(J(t))$  is not a convergent integral because  $J(t)$  grows with increasing time
  - Solution<sup>2</sup>:
    - $J(t)$  grows with increasing time but its second derivative vanishes at large time
    - Complex modulus is related to the Fourier transform of the creep compliance second derivative

<sup>1</sup>Findley, 1976

<sup>2</sup>Evans et al. Physical Review, 2009.

# Acoustic radiation force creep

- Besides shear wave excitation, acoustic radiation force has been used to study tissue steady-state response, assuming that the force is a temporal step function



# Acoustic radiation force creep

- Purpose is to use acoustic radiation force to induce tissue creep response
- Use time-creep compliance conversion formula to get complex modulus → model-free method

$$\text{Creep Compliance: } J = \frac{\varepsilon}{\sigma} = \beta \cdot u(t)$$

$$\therefore G^*(\omega) = \frac{1}{(i\omega) FT[J(t)]} \quad G(\omega) = \frac{1}{\beta} \frac{1}{(i\omega) FT[u(t)]}$$

- Output from **conversion formula** (estimated modulus, C) is scaled by a factor  $\beta$  of the complex modulus, G,

$$C^*(\omega) = \beta \left[ G_s(\omega) + iG_l(\omega) \right] \quad \tan(\delta) = \frac{\beta \cdot G_l(\omega)}{\beta \cdot G_s(\omega)}$$



# Calibrate complex modulus with SDUV

- The wavenumber  $k$  and the shear modulus  $G$  are simply linked through the shear wave propagation equation

$$G = \rho \frac{\omega^2}{k^2}$$

$\rho$  = density

$\omega$  = frequency

- In the case of linear viscoelastic medium, the shear modulus is complex,  $G = G_s + iG_l$ , and the wavenumber is complex,  $k = k_r + ik_i$ , then:

$$G_s(\omega) = \rho\omega^2 \frac{k_r^2 - k_i^2}{(k_r^2 + k_i^2)^2}$$

$c_s$  = shear wave speed

$k_r = \omega/c_s$

$\alpha$  = shear wave attenuation

$k_i = \alpha$

$$G_l(\omega) = -2\rho\omega^2 \frac{k_r k_i}{(k_r^2 + k_i^2)^2}$$

# RF Creep and shearwave relation

- From radiation force creep, we can get the loss tangent or the ratio between  $G_l$  and  $G_s$
- From shear wave dispersion, we can get the real wave number,  $k_r$  ( $k_r = \omega/c_s$ ).
- Then, if we know  $\tan(\delta)$  and  $k_r$ , we can estimate  $k_i$  (shear wave attenuation  $\alpha$ )

$$\frac{G_s(\omega)}{G_l(\omega)} = \frac{k_r^2 - k_i^2}{2k_r k_i} \quad k_i = k_r \left( \frac{1}{\tan(\delta)} - \sqrt{1 + \left( \frac{1}{\tan(\delta)} \right)^2} \right)$$

- If both  $k_r$  and  $k_i$  are known, we can get the complex modulus  $G^*$

## Materials and Method

- Two homogeneous elasticity phantoms (custom-made by CIRS, Inc., Norfolk, VA) and one excised swine kidney were used in this study.
- A Verasonics V-1 ultrasound system (Verasonics, Redmond, WA) equipped with a L7-4 linear array transducer.
- Creep displacement is induced by acoustic radiation force to estimate  $\tan(\delta)$  and shear wave dispersion ultrasound vibrometry is used to calculate the model-free complex shear modulus



# Materials and Method

## Model-Free modulus

$$G_s(\omega) = \rho\omega^2 \frac{k_r^2 - k_i^2}{(k_r^2 + k_i^2)^2}$$

$$G_l(\omega) = -2\rho\omega^2 \frac{k_r k_i}{(k_r^2 + k_i^2)^2}$$

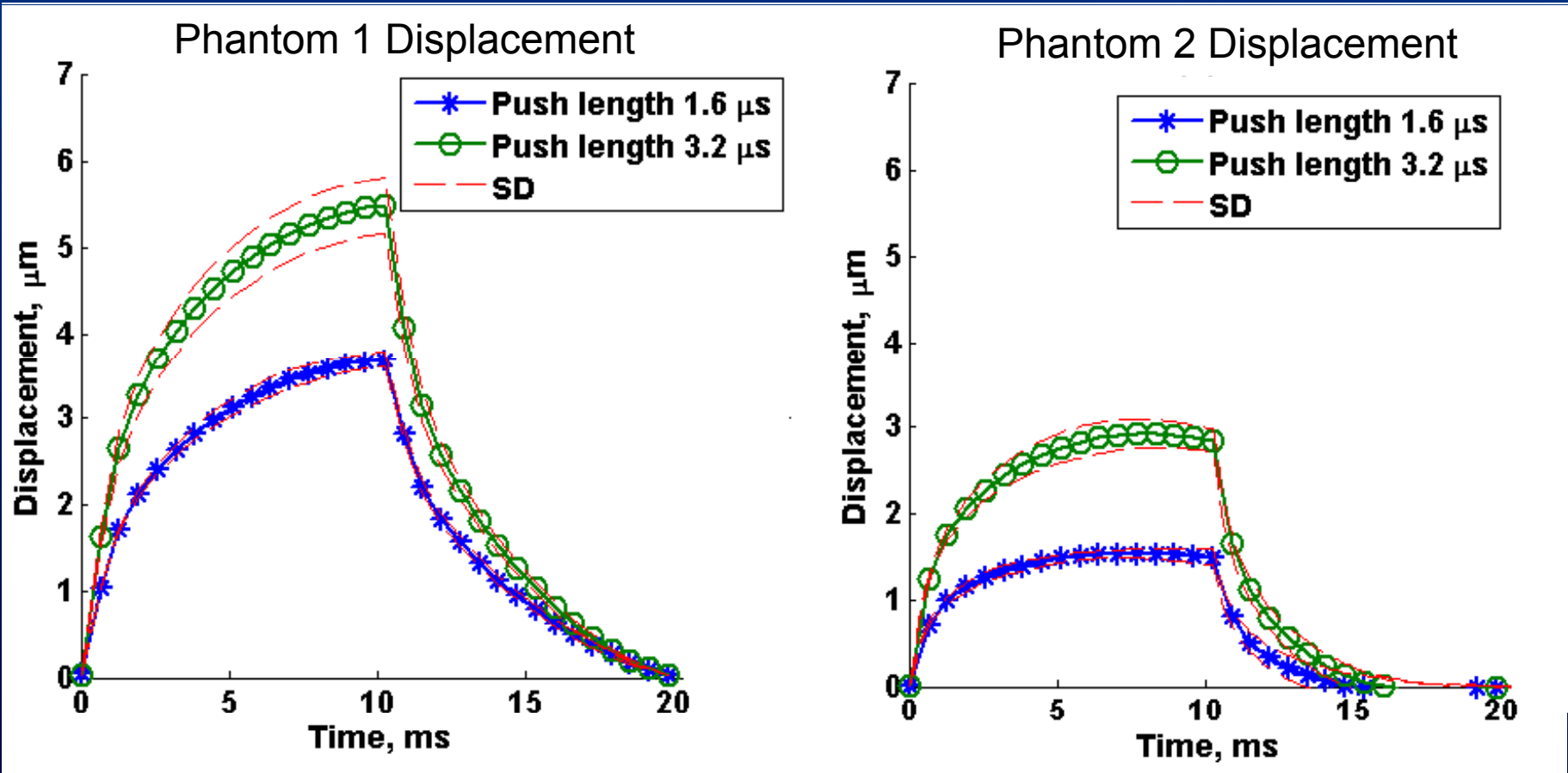
## Radiation force creep

$$k_i = k_r \left( \frac{1}{\tan(\delta)} - \sqrt{1 + \left( \frac{1}{\tan(\delta)} \right)^2} \right)$$

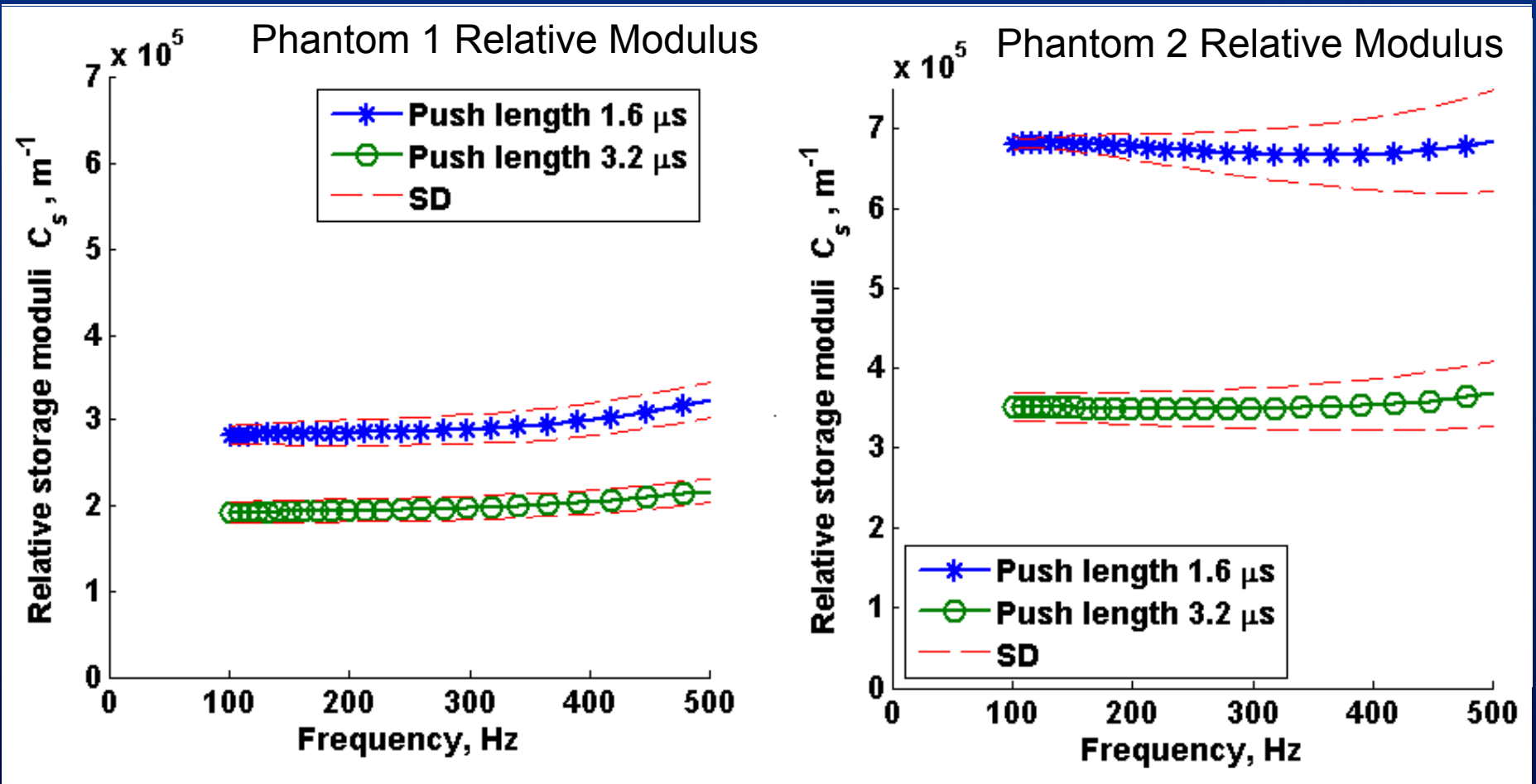
## Shear wave

$$k_r = \omega/c_s$$

# Results – Creep Displacement

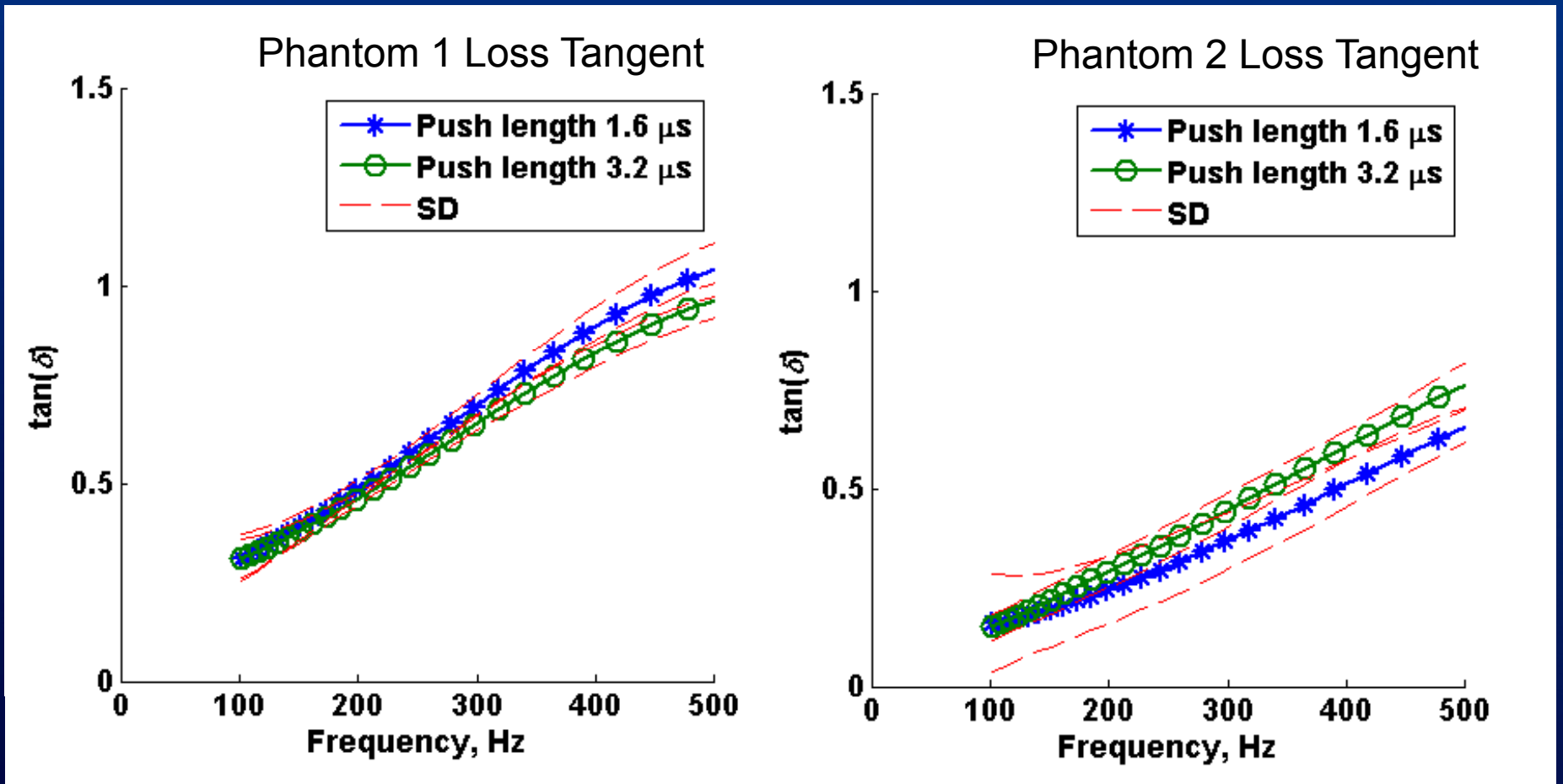


# Results – Relative Modulus

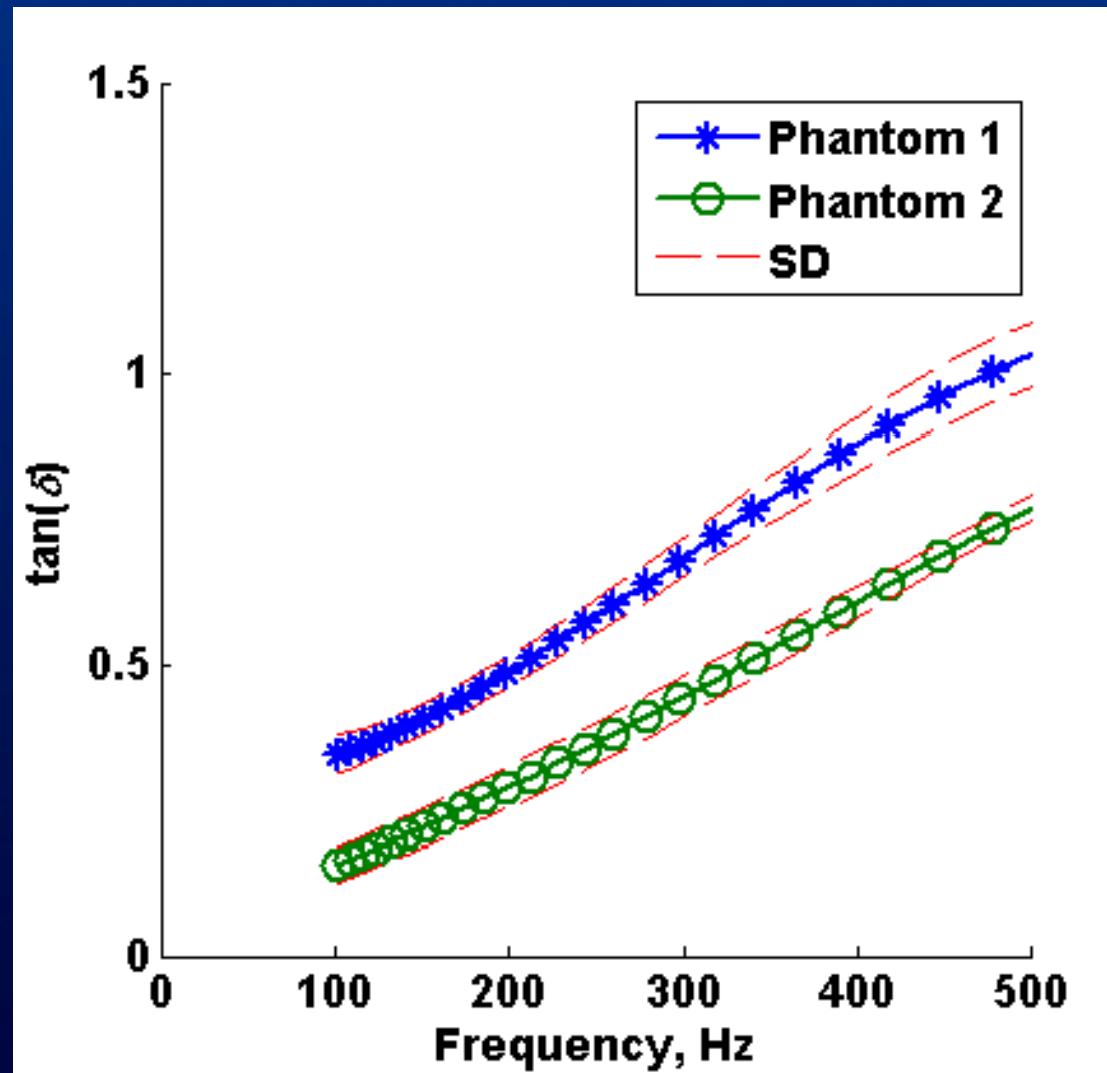




# Results – Loss Tangent

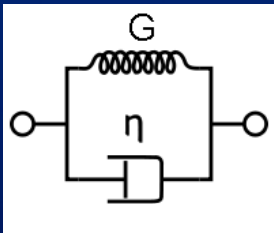


# Results – Loss Tangent



# Results Phantom 1

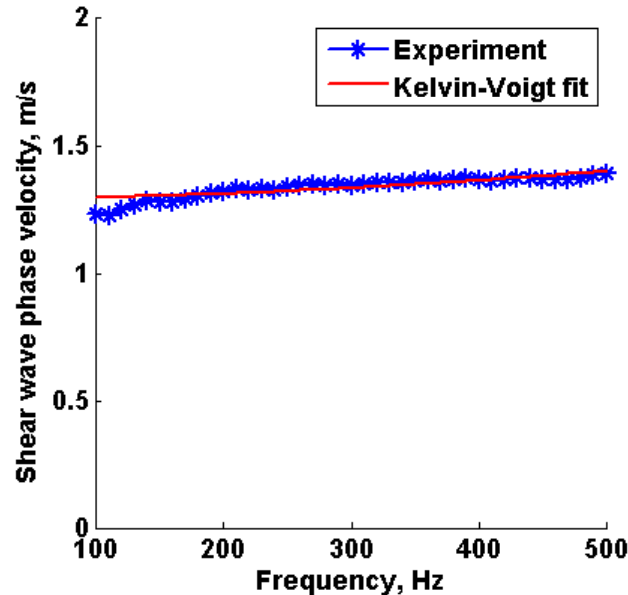
Kelvin-Voigt



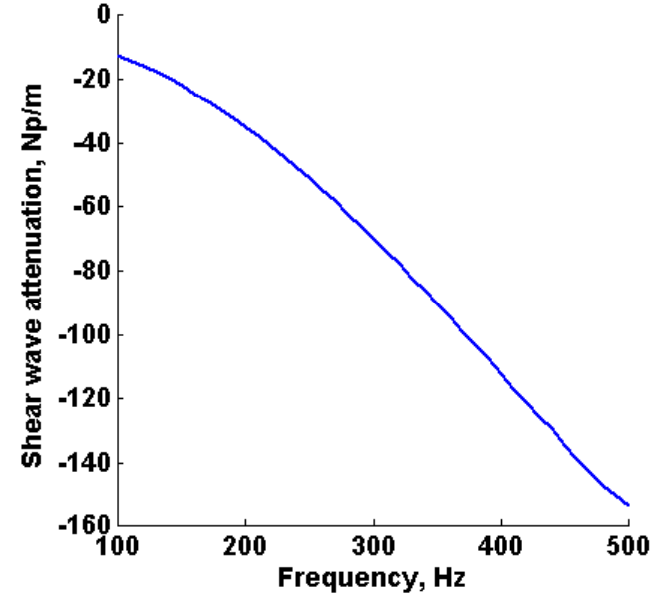
$$G^*(\omega) = G + i\omega\eta$$

$G^*$  = Complex shear modulus  
 $G$  = Shear modulus  
 $\eta$  = Viscosity

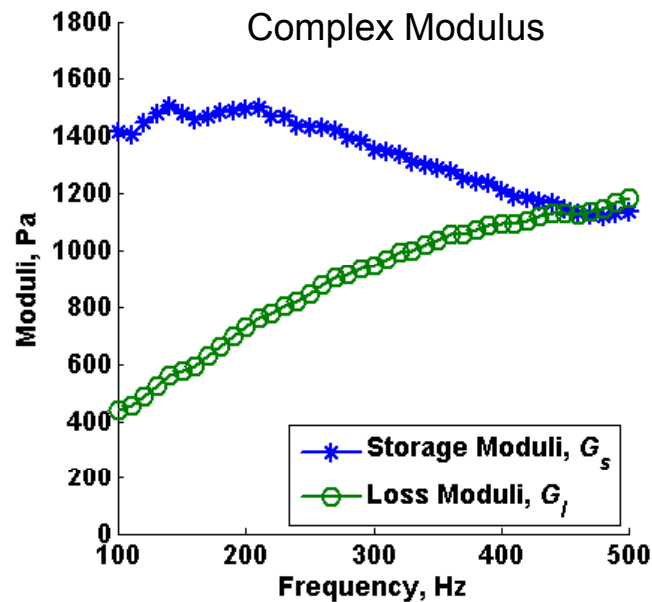
Shear Wave Velocity Dispersion



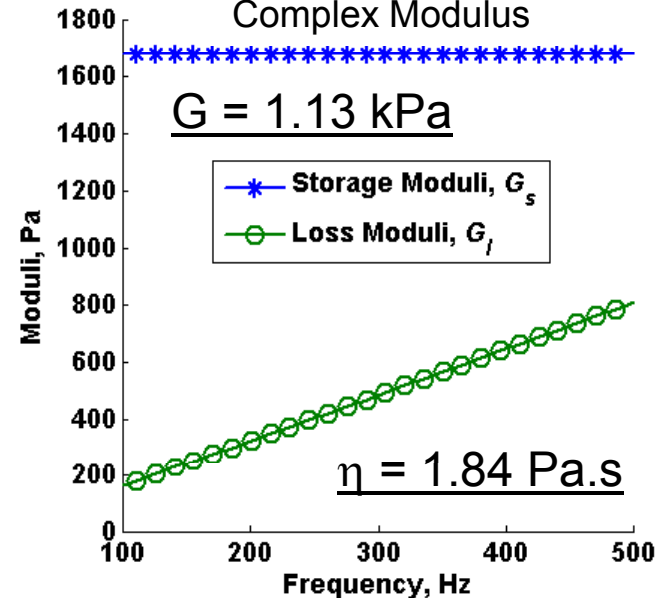
Shear Wave Attenuation



Model-free  
Complex Modulus



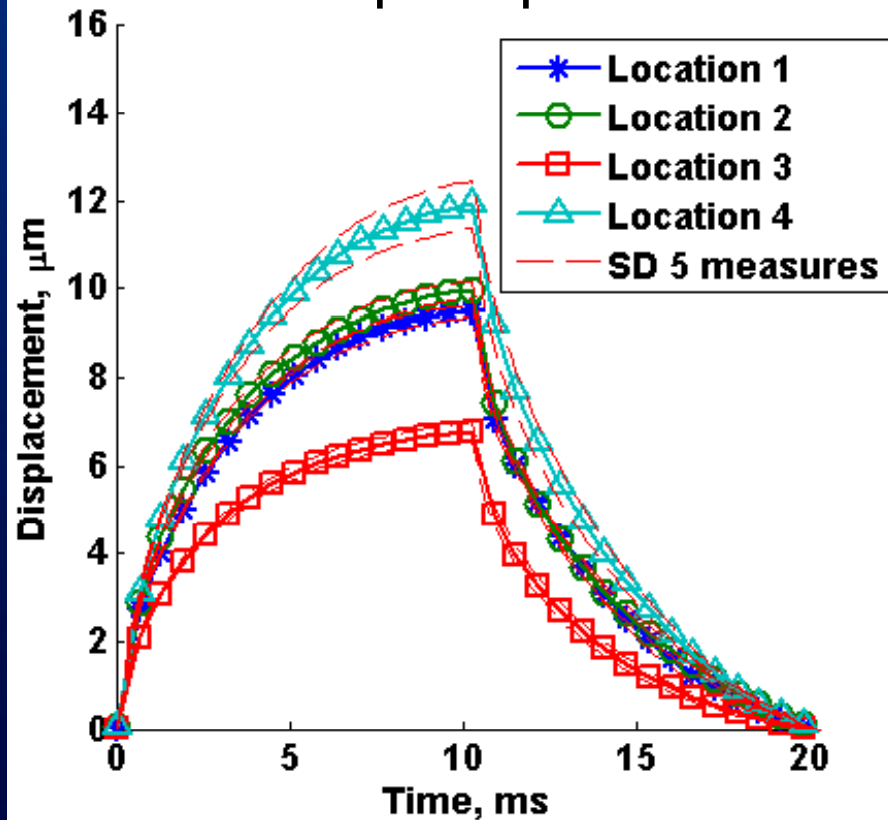
Voigt Model-based  
Complex Modulus



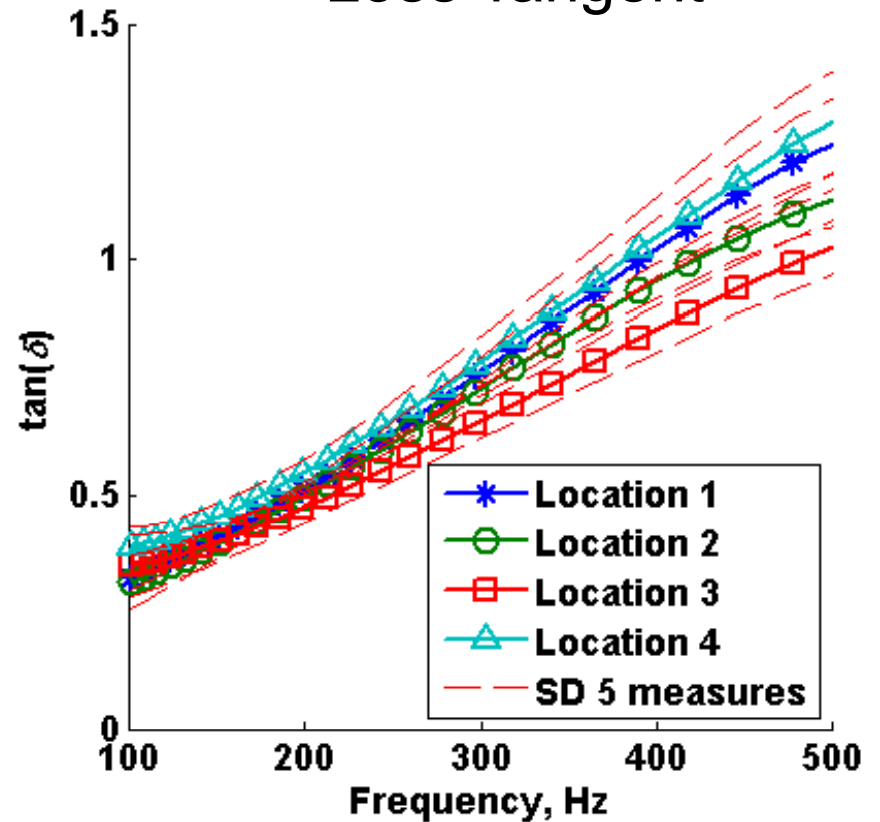


# Results – Excised Kidney

## Creep Displacement

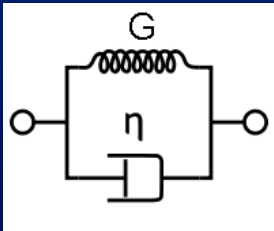


## Loss Tangent



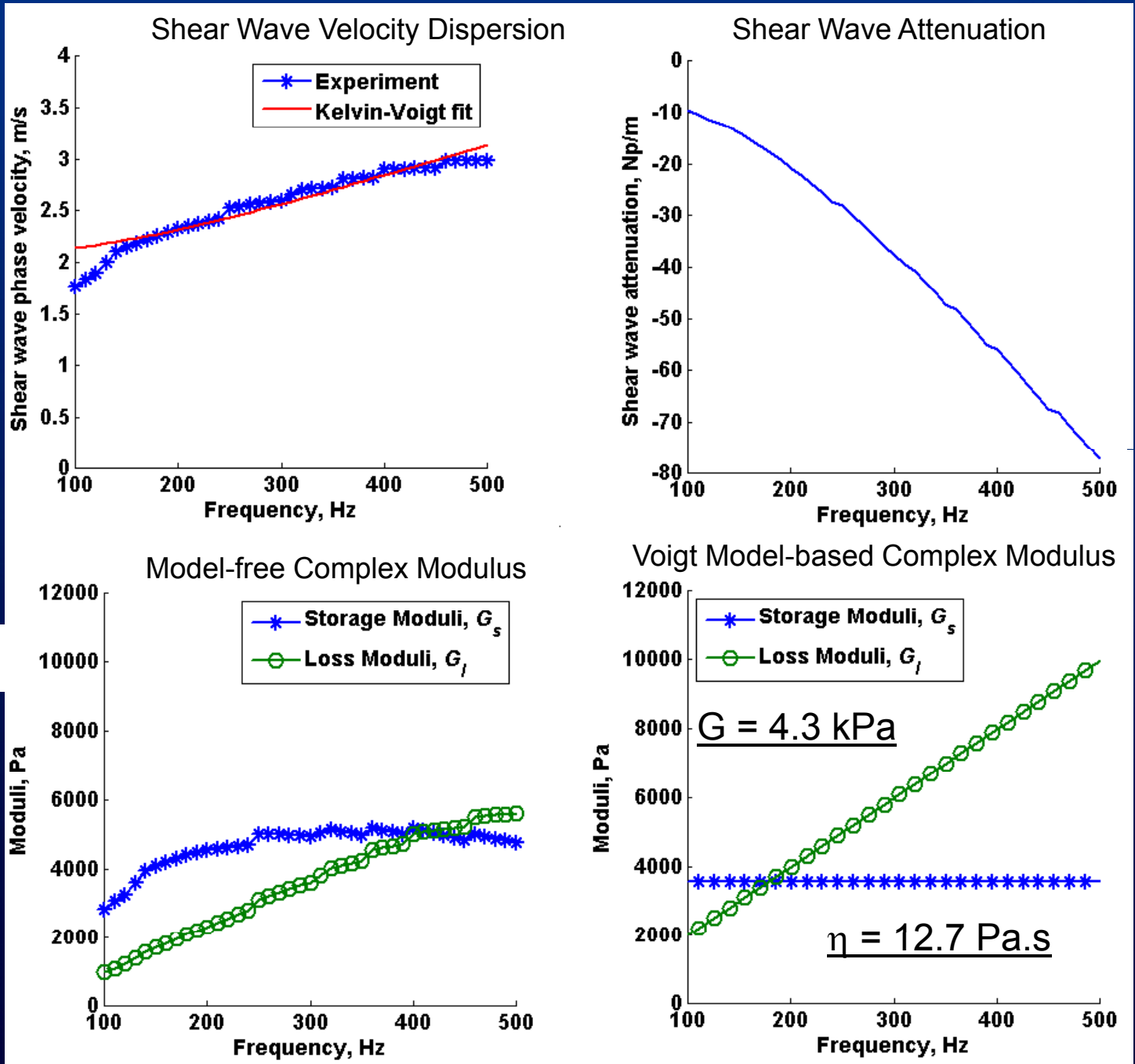
# Results Excised Kidney

Kelvin-Voigt



$$G^*(\omega) = G + i\omega\eta$$

$G^*$  = Complex shear modulus  
 $G$  = Shear modulus  
 $\eta$  = Viscosity



# Conclusion

- Presented a model to measure viscoelastic properties by studying creep response induced by acoustic radiation force
- Advantages:
  - Model free!
  - Fast acquisition (10 ms), local measurements ( $3 \times 1 \text{ mm}^2$ )
  - Measurements over a wide frequency range with high resolution
    - Low frequencies could be explored if creep is maintained for longer periods
  - Robust approach to estimate complex modulus by using the analytic solution to the complex compliance vs. modulus constitutive equation
  - Push beams are compatible with Doppler pulse, therefore this method is compatible with most ultrasound scanners.

## Acknowledgments

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# Mayo Clinic Ultrasound Research Laboratory

